MAT 1475
Fall 2014
Professor K. Poirier
Test #2
October 29, 2014

Name SOLUTIONS

Time Limit: 100 Minutes

This exam contains 6 pages and 12 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use your calculator. No other aids are allowed.

Total: 50 points, including 2 points overall for clear, precise, effective, and complete communication and justification (this includes correct use of all symbols and notation)

Style:

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1. (5 points) Use the **limit definition** of the derivative to differentiate the following function.

$$f(x) = 2x^2 - 5x \qquad \leftarrow \text{so } f(x+h) = 2(x+h)^2 - 5(x+h)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h - 2x^2 + 5x}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$$

$$= \lim_{h \to 0} 4x + 2h - 5$$

$$= 4x + 2 \cdot 0 - 5$$

$$= 4x - 5$$

2. (5 points) Differentiate. [redacted] simplify your answer.

$$f(x) = \frac{4x^2 - 5}{x^2} = 4 - 5x^{-2}$$

$$f'(x) = 0 - 5(-2)x^{-3}$$

$$= \frac{10}{x^2}$$

3. (5 points) Differentiate. ~~Do not~~ simplify your answer.

$$f(x) = \frac{4x^2 - 5}{x^2 - 1}$$

quotient rule:

$$f'(x) = \frac{(8x)(x^2-1) - (4x^2-5)(2x)}{(x^2-1)^2}$$

$$= \frac{8x^3 - 8x - 8x^3 + 10x}{(x^2-1)^2}$$

$$= \frac{-8x + 10x}{(x^2-1)^2}$$

4. (5 points) Differentiate. Do not simplify your answer.

$$f(x) = (7\tan(3x) + 5x^2)^4$$

chain rule:

$$f'(x) = 4\left(7\tan(3x) + 5x^2\right)^3 \left(7\sec^2(3x)\cdot 3 + 10x\right)$$

5. (5 points) Differentiate. Do not simplify your answer.

product rule :

$$f(x) = \sin(7x)\ln(x^5)$$

$$f'(x) = \cos(7x)\cdot 7\cdot \ln(x^5) + \sin(7x)\frac{1}{x^5}\cdot 5x^4$$

$$= 7\cos(7x)\ln(x^5) + \frac{5\sin(7x)}{x}.$$

6. (5 points) Find the equation of the line tangent to the graph of $f(x) = 2x^3 + 5x^2 + 6$ at the point with $x = -1$. Leave your answer in slope-y-intercept form.

Equation of line: $y - y_1 = m(x - x_1)$

$$\underset{f(-1)}{\underset{\uparrow}{y_1}} \quad \underset{f'(-1)}{\underset{\uparrow}{m}} \quad \underset{-1}{\underset{\uparrow}{x_1}}$$

$$f(-1) = 2(-1)^3 + 5(-1)^2 + 6$$
$$= -2 + 5 + 6$$
$$= 9$$

$$f'(x) = 6x^2 + 10x$$
$$f'(-1) = 6(-1)^2 + 10(-1)$$
$$= 6 - 10$$
$$= -4$$

so the equation is

$$y - 9 = -4(x - (-1))$$
$$y = -4x - 4 + 9$$
$$y = -4x + 5$$

7. (5 points) Find the equation of the line tangent to the graph of $f(x) = \log_5(2x^2 + 7)$ at the point with $x = 3$. Leave your answer in slope-y-intercept form.

equation of line: $\underset{f(3)}{y - y_1} = \underset{f'(3)}{m}(x - \underset{3}{x_1})$

$f(3) = \log_5(2 \cdot 3^2 + 7)$
$= \log_5(18 + 7)$
$= \log_5(25)$
$= 2$

$f'(x) = \frac{1}{\ln(5)(2x^2+7)}(4x)$

$f'(3) = \frac{12}{\ln(5) \cdot 25}$

so the equation of the line is

$$y - 2 = \frac{12}{\ln(5) \cdot 25}(x - 3)$$

$$y = \frac{12}{\ln(5) \cdot 25}x - \frac{36}{\ln(5) \cdot 25} + 2$$

8. (5 points) Find the fourth derivative of the function.

$$f(x) = e^x \sin(x)$$

product rule

$$f'(x) = e^x \sin(x) + e^x \cos(x)$$
$$= e^x(\sin(x) + \cos(x))$$

$$f''(x) = e^x(\sin(x) + \cos(x)) + e^x(\cos(x) - \sin(x))$$
$$= e^x(\sin(x) + \cos(x) + \cos(x) - \sin(x))$$
$$= 2e^x \cos(x).$$

$$f'''(x) = 2e^x\cos(x) - 2e^x\sin(x)$$
$$= 2e^x(\cos(x) - \sin(x))$$

$$f^{(4)}(x) = 2e^x(\cos(x) - \sin(x)) + 2e^x(-\sin(x) - \cos(x))$$
$$= 2e^x(\cos(x) - \sin(x) - \sin(x) - \cos(x))$$
$$= -4e^x \sin(x).$$

9. (5 points) A student threw her calculus book out a window of height ~~50~~ 100 meters. After t seconds, the book has fallen $4.9t^2$ meters. Determine how fast the book was traveling when it hit the ground.

When the book hits the ground, it has fallen 100 m.
so we need to solve $4.9t^2 = 100.$

$$t^2 \simeq 20.40816$$
$$t \simeq \pm 4.51754$$

We'll disregard the negative value of t and use $t \simeq 4.51754$ s.
The instantaneous velocity at time t is given by $\frac{d}{dt}(4.9t^2)$
$$= 9.8t$$
so at $t = 4.51754$, its velocity $\simeq (9.8)(4.51754)$
$$= 44.271892 \text{ m/s}.$$

TRUE/FALSE - YOU DO NOT NEED TO JUSTIFY YOUR ANSWER

10. (1 point) Determine whether the following statement is true or false. You do not need to justify your answer.

The derivative of a linear function is not always a constant function.

Circle one: true false

(if $f(x) = mx+b$, then $f'(x) = m$ is always constant).

FILL IN THE BLANK:

11. (1 point) Write our class's slogan for derivatives:

The derivative is a formula for the slope of the tangent line.

GRAPHING

12. (1 point) Consider the graph of the function $f(x)$. Graph its derivative $f'(x)$.