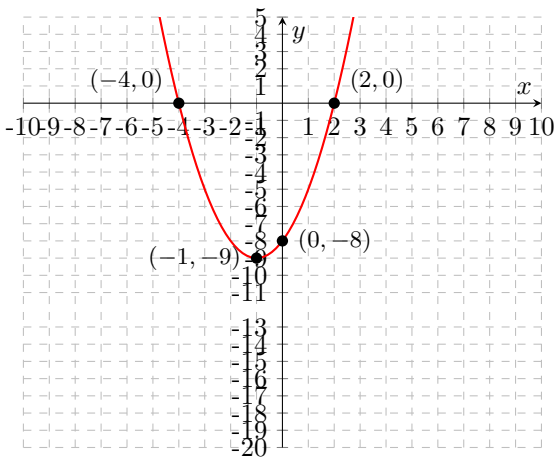
Final Exam Review- MAT 1275 - Fall 2023

The following are the types of problems you may see on the final exam (they are not necessarily the exact problems).

1 Questions requiring supportive work

1. Simplify $\dfrac{x^2}{x^2-4} - \dfrac{x-2}{x+2}$.
2. Simplify $\left(\dfrac{4x^3y^{-2}}{16x^{-3}y^4}\right)^{-1/2}$.
3. Write in the form $a+bi$: $\dfrac{2-5i}{3-4i}$.
4. Simplify $(3\sqrt{5}+2\sqrt{7})(3\sqrt{5}-2\sqrt{7})$.
5. Simplify $\dfrac{9x^4y^3 - 12x^2y^4 + 3xy^3}{3xy^3}$.
6. Simplify $\dfrac{\dfrac{x^2-4}{x^2-x}}{\dfrac{x^2-3x+2}{x}}$.
7. Write using one radical $\sqrt[3]{\sqrt{x^5}}$.
8. Solve for x and simplify the answer: $2x^2 - 2x - 3 = 0$.
9. Solve $\dfrac{x}{x-2} = -\dfrac{1}{3} + \dfrac{10}{x}$.
10. Solve for y and simplify the answer: $\sqrt{y-2} - 4 = -y$.
11. Evaluate $x^3 - 2x^2 - 5x + 6$ at $x = 1$. What does this tell you about $x^3 - 2x^2 - 5x + 6$? Use long division to rewrite this polynomial as a product of two factors and then find all solutions to the equation $x^3 - 2x^2 - 5x + 6 = 0$.
12. Find all solutions to the equation $3x^3 - 27x = 0$.
13. Find an equation for a line which passes through $(1, -5)$ which is perpendicular to $y = \dfrac{1}{3}x + 4$. Graph your line.
14. Given the following quadratic equation, sketch and label the graph: $y^2 = -x^2 - 4x + 5$.
15. By completing the square, find the vertex and the axis of symmetry of the parabola whose equation is the following: $y = x^2 - 4x + 5$.
16. (This problem doesn't require work but the answer is included below.) Consider the following graph of an equation of the form $y = ax^2 + bx + c$.



Mark true or false.

(a) There is a solution for which $y = -9$.

(b) There is exactly one solution for which $x = \sqrt{3}$.

(c) The value of c is -9.

(d) The value of a is negative.

(e) The system $\begin{cases} y &= ax^2 + bx + c \\ y &= 3 \end{cases}$ has exactly two solutions.

(f) The axis of symmetry is given by the equation $y = -9$.

17. Find all solutions to
$$\begin{cases} 2x - 3y^2 = 1 \\ -x + 2y = -4 \end{cases}$$

18. Find all solutions to
$$\begin{cases} 2x - 3y = 7 \\ -x + 2y = -4 \end{cases}$$

19. A 10-foot ladder is leaning up against a wall. If the base of the ladder is situated 4 feet away from the base of the wall, what is the angle of elevation of the ladder? Draw a picture that depicts the situation and label the relevant information. Round your answer to the nearest tenth of a degree.

20. For each of the two expressions $\cos(-210°)$ and $\tan\left(\frac{7\pi}{4}\right)$:

    (a) identify the quadrant in which the angle is located,

    (b) find the reference angle,

    (c) find the exact value.

21. Given $\sin(x) = -\frac{2}{5}$ and $\cos(x) > 0$, find the other 5 trigonometric values.

22. Given a ΔABC with $\angle C = 40°$, $c = 3$ feet and $b = 4$ feet, find $\angle A$. Draw a picture of the triangle and label it with the information provided. Round each answer to the nearest tenth. You may use either the law of sines:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
or the law of cosines:
$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos(C) \\ a^2 &= c^2 + b^2 - 2cb\cos(A) \\ b^2 &= a^2 + c^2 - 2ac\cos(B). \end{aligned}$$

23. Solve for x and round your answer to the nearest tenth:
$$3^x = 12.$$

24. Evaluate (without using a calculator):
$$\log_5(25\sqrt[4]{5}).$$

25. Evaluate $-2^{-4} + 3^0 - (2/3)^{-1}$ and indicate where it is on a number line.

26. Simplify $\dfrac{\sqrt[4]{x^3}}{\sqrt{x}}$.

2 True/False questions

The following is just a sample of the types of problems you may find. Label the following True or False.

1. There could be 5 solutions to the system of equations: $\begin{cases} x^2 + y^2 = 6 \\ x^2 - y = 8 \end{cases}$
2. The product of two polynomials of degree 3 has degree 9.
3. If 7 is a root of a polynomial, then $(x - 7)$ is a factor of that polynomial.
4. The lines $y = \frac{1}{2}x - 9$ and $y = 2x - \frac{1}{2}$ are perpendicular.
5. If $\cos\theta < 0$, then θ is either in the second or the third quadrant.
6. The equation $3^x = 12$ has one solution and it is between 2 and 3.
7. $\log 7$ is bigger than 2.
8. The expression $\sqrt{9x^9y^4}$ is equal to $3x^3y^2$ for all positive x and y.
9. The equations $\frac{1}{2}x^2 + \frac{1}{3}x = \frac{1}{6}$ and $3x^2 + 2x = 1$ are equivalent (they have the same solutions).
10. $(a + b)^2 = a^2 + b^2$
11. $(2\sqrt{7} - 3\sqrt{5})(2\sqrt{7} + 3\sqrt{5}) = 4 \cdot 7 - 9 \cdot 5$.
12. $x^{-1} = -x$.
13. $(2\sqrt{7})^2 = 2 \cdot 7 = 14$.

3 Give example questions

The following are just examples of the types of examples you may be asked to provide. Give an example of:

(a) an equation of a line that passes through $(2, 5)$

(b) a radical expression

(c) a rational equation in one variable for which -3 cannot be a solution

(d) a polynomial with leading coefficient -7, degree 5, and 4 terms

(e) a polynomial of degree 2 with roots -3 and 5

(f) an equation of a horizontal line that passes through $(8, -\sqrt{3})$

(g) an equation of a circle with center in the second quadrant

(h) trigonometric ratio which has a different sign than tangent of an angle in quadrant III.

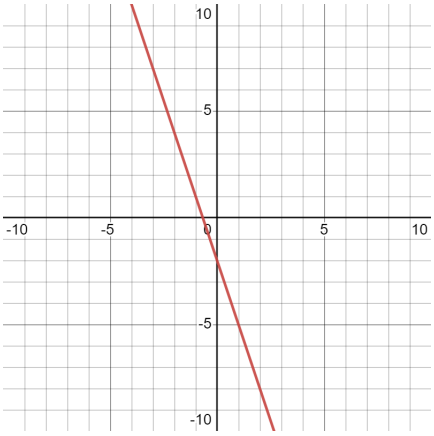
(i) an equation that has no real solution

(j) an equation with one variable that has two solutions

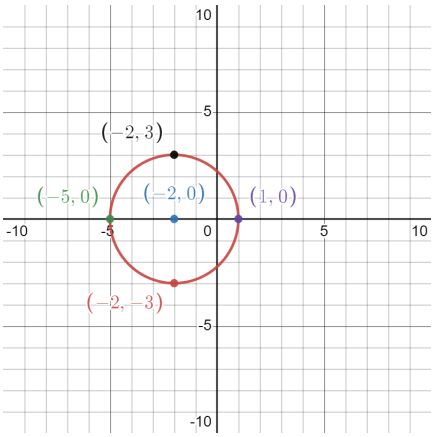
(k) Give an example of how to add fractions with unequal denominators.

4 Answers to questions from part 1

1. $\dfrac{4(x-1)}{(x+2)(x-2)}$
2. $\sqrt{\dfrac{3y^6}{2x^6}}$
3. $\dfrac{26}{25} - \dfrac{7}{25}i$
4. 17
5. $3x^3 - 4xy + 1$
6. $\dfrac{x+2}{(x-1)^2}, x \neq 2$
7. $\sqrt[6]{x^5}$
8. $\dfrac{1}{2} \pm \dfrac{\sqrt{7}}{2}$
9. $x = 3$ or 5
10. $y = 3$
11. $x = -2, 1$, or 3
12. $x = 0, 3$, or -3
13. $y + 5 = -3(x-1)$



14. This equation is equivalent to $(x+2)^2 + y^2 = 9$ and so the solutions form a circle centered at $(-2, 0)$ with radius 3:



15. $y - 1 = (x-2)^2$, The vertex is $(2, 1)$ and the axis of symmetry is $x = 2$.
16. True, True, False, False, True, False
17. $(2, -1), (\dfrac{26}{3}, \dfrac{7}{3})$

18. $(2, -1)$

19. $\cos^{-1} \dfrac{4}{10} \approx 66.4°$

20. II, $30°$, $\dfrac{-\sqrt{3}}{2}$; IV, $\dfrac{\pi}{4}$,-1

21. $\cos(x) = \dfrac{\sqrt{21}}{5}, \tan(x) = \dfrac{-2}{\sqrt{21}}, \csc(x) = \dfrac{-5}{2}, \sec(x) = \dfrac{5}{\sqrt{21}}, \cot(x) = \dfrac{-\sqrt{21}}{2}$

22. $81°$

23. 2.3

24. $\dfrac{9}{4}$

25. $-\dfrac{9}{16}$, which is between -1 and 0 on the number line.

26. $\sqrt[4]{x}$.