MAT1575 Module 4 – Taylor Polynomials.

Objectives: To compute and graph Taylor polynomial approximations using Desmos.

1. Compute $p_5(x)$ at the given a value for the following functions.
   (a) $f(x) = \sin(x)$ at $a = 0$
   (b) $f(x) = \frac{1}{1-x}$ at $a = 0$ (Hint: Think about the geometric series from MAT1375.)
   (c) $f(x) = \ln(1-x)$ at $a = 0$ (Hint: Compare the first derivative of this problem to the previous one.)
2. Compute $p_2(x)$ for $f(x) = \sqrt{x+3}$ at $a = 1$.
3. Use Desmos to complete the following tasks.
   (a) Plot $\ln(1-x)$ and $p_5(x)$ at $a = 0$. (Hint: To plot $p_5(x)$, define $f(x)$ as $\ln(1-x)$ and use $f'(x), f''(x)$, etc to compute its derivatives.)
   (b) Highlight the set of points (x, y) such that $|\ln(1-x) - y| < 0.5$. (Hint: $|x-y| < 0.5$ means that $x - 0.5 < y$ and $y < x + 0.5$.)
   (c) For what values of x is the approximation $p_5(x)$ of $\ln(1-x)$ within 0.5 of the right answer?
   (d) Use $p_5(x)$ to estimate the value of $\ln(2)$. (Hint: A smart choice of x and the fact that $-\ln(x) = \ln(\frac{1}{x})$ means that you can get an answer that is within 0.01 of the right value.)