

Name: _____

Points: _____

1. The Greatest Common Factor

The **greatest common factor (GCF)** for a polynomial is the largest monomial that divides each term of the polynomial. GCF contains the GCF of the numerical coefficients and each variable raised to the smallest exponent that appear.

a. Factor $8x^4 - 4x^3 + 16x^2 - 4x + 24$

b. Factor $9a^3b^4 - 3a^2b^3 + 6ab^2$

2. Factoring by Grouping

a. Factor $56 + 21k + 8h + 3hk$

b. Factor $5x^2 + 40x - xy - 8y$

3. Factoring Trinomials with Lead Coefficients of 1**Steps to factoring trinomials with lead coefficient of 1**

1. Write the trinomial in descending powers.
2. List the factorizations of the third term of the trinomial.
3. Pick the factorization where the sum of the factors is the coefficient of the middle term.
4. Check by multiplying the binomials.

a. Factor $x^2 - 8x + 12$

b. Factor $y^2 - 9y - 36$

c. Factor $a^2 + 7ab - 8b^2$

d. Factor $x^2 - 9xy + 18y^2$

e. Factor $x^3 - 14x^2 + 45x$

f. Factor $2x^2 - 24x - 90$

4. Factoring Trinomials with Lead Coefficients other than 1

Method: Trial-and-error

1. Multiply the resultant binomials.
2. Check the sum of the inner and the outer terms is the original middle term bx .

a. Factor $2h^2 - 5h - 12$

b. Factor $3k^2 - 14k - 5$

5. Factoring Special Products

Many trinomials can be factored by using the following special product formulas:

Factoring Perfect Square Trinomial
$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

a. Factor $p^2 + 10p + 25$

b. Factor $16a^2 - 40a + 25$

c. Factor $9b^2 + 42b + 49$

6. Factoring a Difference of Squares**Difference of Squares formula:** $a^2 - b^2 = (a + b)(a - b)$

a. Factor $x^2 - 49$

b. Factor $36x^2 - 25y^2$

c. Factor $x^2 + 36$

d. Factor completely $x^4 - 81$

7. Factoring a Polynomial

To factor a polynomial, first factor the greatest common factor, then consider the number of terms in the polynomial.

- I. Two terms: Determine if the binomial is a difference of squares.
If it is a difference of squares, then $a^2 - b^2 = (a + b)(a - b)$
- II. Three Terms: Determine if the trinomial is a perfect square trinomial.
 - a) If the trinomial is a perfect square then
$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
 - b) If the trinomial is not a perfect square, then
 - i) If it is $x^2 + bx + c$, then
find two factors $(x + \text{first number})(x + \text{second number})$
 - ii) If it is $ax^2 + bx + c$, then use trial and error or the factoring method.
- III. Four terms: Try to factor by grouping.

a. Factor completely $4x^3y - 12x^2y^2 + 9xy^3$

b. Factor completely $20a - 5a^3$

c. Factor completely $12x^3 + 11x^2 + 2x$

d. Factor completely $7z^2w^2 - 10zw^2 - 8w^2$

5. Solving Quadratic Equations by Factoring

To solve a quadratic equation by factoring:

- 1) Put the equation in standard form: $ax^2 + bx + c = 0$.
- 2) Factor completely.
- 3) Use the zero-product rule, set each factor containing the variable equal to zero and solve for x.
Note: Do not solve for the constant factor.

a) Solve for x: $2x^2 - 18 = 0$

b) Solve for x: $2x^2 - 18x = 0$

c) Solve for x: $2x^3 + 9x^2 = 5x$

d) Solve for x: $x(5x + 2) = 3$