MAT1575 Module 3 – Logistic Growth

Objectives: Construct a logistic growth model for the weight of an animal.

1. Let $w := w(t)$ be the weight of an animal over time. A weight growth model, known as the logistic growth model, is given by the differential equation

$$\frac{dw}{dt} = r \cdot w \left(1 - \frac{w}{K}\right)$$

where r and K are constants. The goal is to find a general solution, that is, function w that satisfies the differential equation given above.

2. The method of separation of variables (a topic in MAT2680 - Differential Equations) says that we can find a general solution by computing both sides of the following equation:

$$\int \frac{1}{w\,(1 - w/K)}\, dw = \int r\, dt.$$

3. Show that

$$\int \frac{1}{w\,(1 - w/K)}\, dw = \ln\left|\frac{w}{K - w}\right| + C.$$

Hint: Use the fact that $\frac{1}{w(1-w/K)} = \frac{K}{w(K-w)}$ and partial fraction decomposition.

4. From parts 2 and 3, we have that

$$\begin{aligned} \ln\left|\frac{w}{K - w}\right| + C &= rt \\ \left|\frac{w}{K - w}\right| &= e^{rt - C} \\ \frac{w}{K - w} &= ae^{rt} \end{aligned}$$

where $a = \pm e^{-C}$. We adjust the sign of a so we can get rid of the absolute value sign.

5. Solve for w in part 4 to get that

$$w = \frac{K}{1 + Ae^{-rt}}$$

where $A = 1/a$.

6. Go to `https://www.desmos.com/calculator/qkoahg0ecu` to see a table of values for a dog's weight over time[1].

[1] Original data can be found here: `https://bscheng.com/2014/05/07/modeling-logistic-growth-data-in-r/`.

7. In Desmos, if you have a table of values, you can try to fit a function to the data using the ~ operator. For example, if you type in

```
y_1 ~ m * x_1 + b
```

   Desmos will try to find the "best" line that fits the data (and it will tell you the values of m and b.)

8. Now use part 5 to find the "best" logistic growth model that will fit the data. Can you interpret what the value of K represents?