Name: ___________________________________ Points: ______

1. To simplifying rational expressions to lowest terms

1. Factor the polynomial in the numerator and denominator.
2. Apply the fundamental principle of rational expressions to divide out factors common to both the numerator and denominator.

$$\frac{PR}{QR} = \frac{P}{Q} \quad \text{where } R \neq 0$$ [Fundamental Principle of Rational Expressions]

a) Simplify: $\frac{p+2}{p+2}$

b) Simplify: $\frac{p+2}{2+p}$

c) Simplify: $\frac{p-2}{p-2}$

d) Simplify: $\frac{p-2}{2-p}$

e) Simplify: $\frac{2y^2-8y}{3y^2-12y}$

2. Multiplying Rational Expressions

Recall in fractions: $\frac{12}{25} \bullet \frac{15}{16} = \frac{3}{5} \bullet \frac{3}{4} = \frac{9}{20}$

To Multiply Rational Expressions:

1. Factor all the polynomials in the numerators and denominators.
2. Multiply the numerators and multiply the denominators.
3. Simplify the product by applying the fundamental property and dividing the numerator and denominator by their common factors.

a) Multiply: $\frac{12}{18} \cdot \frac{24}{6}$

b) Multiply: $\frac{15a^4}{14a^5b} \cdot \frac{21b^5}{25ab}$

Prepared by Profs. Ghezzi, Han, and Liou-Mark
Supported by BMI, NSF STEP #0622493, and Perkins

c) Multiply: $\dfrac{x^2-64}{3+x}\cdot\dfrac{x+3}{x^2-8x}$

d) Multiply: $\dfrac{12a^2-18a}{4a^2+4a+1}\cdot\dfrac{4a^2+8a+3}{4a^2-9}$

3. **Dividing Rational Expressions**

Recall in fractions: $\dfrac{12}{25}\div\dfrac{16}{15}=\dfrac{12}{25}\bullet\dfrac{15}{16}=\dfrac{3}{5}\bullet\dfrac{3}{4}=\dfrac{9}{20}$

To Divide Rational Expressions:

1. Change the division sign to a multiplication sign.
2. Take the reciprocal of the second rational expression (flip the rational expression)
3. Follow the steps used in multiplying rational expressions.

a) Divide: $\dfrac{m^6n^3}{25m^4n}\div 4m^5n^4$

b) Divide: $\dfrac{x^2-36}{4x-x^2}\div\dfrac{3x-18}{x^2-16}$

c) Multiply and divide: $\dfrac{3x^2-5x-2}{x^2+x-2}\cdot\dfrac{x^2+4x-5}{12x^2+7x+1}\div\dfrac{5x^2-9x-2}{8x^2-2x-1}$

Prepared by Profs. Ghezzi, Han, and Liou-Mark
Supported by BMI, NSF STEP #0622493, and Perkins

4. Adding and Subtracting Rational Expressions

1. Adding or subtracting Rational Expressions with the Same Denominators

Recall in fractions if the denominators are the same: $\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}=\frac{4}{8}=\frac{1}{2}$

To add rational expressions with the same denominators:

1. Add or subtract the numerators.
2. Write the result over the common denominator.
3. Simplify the rational expression, if necessary.

a) Add: $\frac{5}{21y}+\frac{4}{21y}$

b) Add: $\frac{3x+11}{x+2}+\frac{x-3}{x+2}$

c) Subtract: $\frac{7r-1}{r^2}-\frac{6r-5}{r^2}$

d) Subtract: $\frac{x^2}{x-5}-\frac{25}{x-5}$

2. Finding the Least Common Denominator (LCD) of Rational Expressions

To find the Least Common Denominator (LCD) of rational expressions:

1. Factor each denominator completely.
2. The least common denominator (LCD) is the product of all unique factors each raised to the greatest power that appears in any factored denominator.

3. Adding and Subtracting Rational Expressions with Different Denominators

Recall in fractions if the denominators are different: $\frac{1}{3}+\frac{3}{4}=\frac{1}{3}\left(\frac{4}{4}\right)+\frac{3}{4}\left(\frac{3}{3}\right)=\frac{4}{12}+\frac{9}{12}=\frac{13}{12}=1\frac{1}{12}$

Prepared by Profs. Ghezzi, Han, and Liou-Mark
Supported by BMI, NSF STEP #0622493, and Perkins

To add and subtract rational expressions with different denominators

1. Find the LCD of the rational expressions.
2. Write each rational expression as an equivalent rational expression whose denominator is the LCD found in step 1.
3. Add or subtract the numerators and write the result over the common denominator.
4. Simplify the resulting rational expression.

a) $\frac{7}{5b^4} - \frac{6}{4b^2}$ LCD = ______________

b) $\frac{3}{2y+10} + \frac{8}{3y-15}$ LCD = _________________

c) $\frac{7}{a^2-a-2} + \frac{a}{a^2+4a+3}$ LCD = ___________

g) $\frac{x+1}{x^2-6x+8} - \frac{3}{x^2-16}$ LCD = ______________

Prepared by Profs. Ghezzi, Han, and Liou-Mark
Supported by BMI, NSF STEP #0622493, and Perkins