Name: ___________________________________Points: ______

1. Properties of Exponents

Multiplication/Product Rule	$x^m \cdot x^n =$
Division/Quotient Rule	$\frac{x^m}{x^n} =$ where $(x \neq 0)$
Zero Exponent	$x^0 =$ where $(x \neq 0)$
Power of a Power	$(x^m)^n =$
Power of a Product	$(x \cdot y)^n =$
Power of a Quotient	$\left(\frac{x}{y}\right)^n =$ where $(y \neq 0)$
Negative Exponent	$x^{-n} =$ where $(x \neq 0)$

Schwartzman (1994) defines **Exponent:** "From the Latin **ex**, meaning 'away, out' and **ponent**, the present participle stem of the word **ponere** meaning 'to put or to place.' When something is **exposed** it is "put out" so it can be seen. Similarly, in mathematical notation the **exponent** is the small number or letter that is "put out" to the right and above the base when that base is being raised to a power, also called a "superscript." An exponent is therefore named after its physical appearance in writing rather than its mathematical significance."

a. Simplify $(mm^2)^3(2m^{-7}n^6)^5$

b. Simplify $\left(\frac{1}{9}\right)^{-1} + \left(-\frac{2}{3}\right)^2 - \left(\frac{3}{6}\right)^0$

Instead of writing out long multiplication of the same number, for example, $2 \times 2 \times 2$, a symbolic representation of this idea was developed, so that $a \times a \times a = a^3$. Rene Descartes introduced this idea in 1637 in his book "La Geometrie," so that any value multiplied by itself a certain number of times could be represented as a^2, a^3, or a^4. Descartes was both a philosopher and a mathematician, and is famous for, among other things, the line, *Cogito, ergo sum*, which in English is *I think, therefore I am*.

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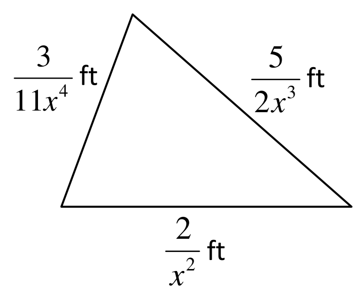
Schwartzman (1994) defines **Power**: "From old French word **poier**, from the Vulgar Latin **potere** a variant of classic Latin **posse** meaning 'to be able.' The Indo-European root is **poti**, meaning 'powerful [as in]: Lord.' If one is able to do many things one is considered powerful. A powerful person typically has a large number of possessions (a word derived from 'posse') and a large amount of money. In algebra when even a small number like 2 (two) is multiplied by itself a number of times, the result becomes very large quickly; metaphorically speaking, the result is powerful. If the term 'power' is used precisely, it refers to the result of multiplying a number by itself a certain number of times."

c. Simplify $\left(\frac{7x^3y^{-4}}{x^{-3}y^9}\right)^3$

d. Simplify $\left(\frac{-6a^{-4}b^6}{a^{-2}b^{-5}c}\right)^{-3}$

2. Adding and Subtracting Rational Expressions

a. Find the perimeter of the triangle:



The Lowest Common Denominator (LCD) = ______________

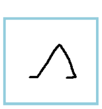


Figure 1

The first known reasoning behind mathematical exponents started with the Egyptians of the Middle Empire, 2040-1630 B.C. (Cajori, 2007). The ancient symbol for squaring a number was found in a hieratic Egyptian papyrus of that period. In part of the ancient Papyrus containing the computation of the volume of a pyramid of a square base occurs a hieratic term containing a pair of walking legs (see Figure 1) signifying "make in going," which means squaring the number.

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b. Add. If possible simplify your answer.

LCD= ___________________________

$$\frac{a-2}{a-4}+\frac{2a^2-15a+12}{a^2-16}$$

c. Subtract. If possible simplify your answer.

LCD= ___________________________

$$\frac{y}{y^2+10y+25}-\frac{4-y}{y^2+6y+5}$$

c. Add. If possible simplify your answer.

LCD= ___________________________

$$\frac{9x+2}{3x^2-2x-8}+\frac{7}{3x^2+x-4}$$

References:
Cajori, F. (2007).*A history of mathematical notations*. Volume 1, 2, 335-339. Chicago, IL: Open Court Publishing Co.

Swartzman, S. (1994).*The words of mathematics: An etymological dictionary of mathematical terms used in English*. USA: The Mathematical Association of America.

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Name: ________________________________ Points: ______

1. Complex Fractions

Simplify.

a. $\dfrac{1-\dfrac{x}{w}}{\dfrac{x^2}{w^2}-1}$ LCD= ______________

b. $\dfrac{\dfrac{2}{x}-\dfrac{x}{2}}{\dfrac{5}{x^2}-9}$ LCD= ______________

c. $\dfrac{\dfrac{2}{a+2}+\dfrac{6}{a+7}}{\dfrac{4a+13}{a^2+9a+14}}$ LCD= ______________

Schwartzman (1994) defines **Fractions**: "From Latin word 'fractus', past participle of the word frangere 'to break' which is the native English conjugate. The Indo-European root is 'bhreg'- of the same meaning. Related borrowings from the Latin includes 'fragile, breakable, diffraction,(breaking up into colors) and fragment.' A fraction is literally a piece broken off something. In fact, in the 16th century English mathematics books referred to fractions as 'broken number'."

2. Fractional Equations

a. What is the difference between $\dfrac{2c}{2c-1}+\dfrac{1}{c}$ and $\dfrac{2c}{2c-1}+\dfrac{1}{c}=\dfrac{1}{2c-1}$?

____

____

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The Babylonians, who lived in the area now known as Iraq , were intensely interested in watching the stars. From these observations, they noticed patterns and deduced that a year – a repetitive pattern with changes in seasons - consisted of 360 days. From this came the idea of fractions, as a year could be divided into days, and days could be multiplied into a year. The use of fractions dates from around 3000 B.C. (Smith, 1958).

b. Solve. $\frac{2c}{2c-1}+\frac{1}{c}=\frac{1}{2c-1}$

LCD= ___________________ $c \neq$ ________

c. Solve. $\frac{2k}{k-3}+\frac{6-2k}{k^2-9}=\frac{k}{k+3}$

LCD= _________________ $k \neq$ _______

d. Solve. $\frac{9}{y^2+7y+10}+\frac{3}{y+5}=\frac{5}{y+2}$

LCD= ___________________ $y \neq$ ________

References:
Smith, D.E. (1958). *History of mathematics*. Vol. 1. New York, NY:. Dover Publications

Swartzman, S. (1994). *The words of mathematics: An etymological dictionary of mathematical terms used in English*. USA: The Mathematical Association of America.

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Name: ___________________________________ Points: ______

1. Rational Exponents

a. The expression $a^{\frac{1}{n}}$ in radical form is written as __________________ .

b. The expression $a^{\frac{m}{n}}$ in radical form is written as __________________ .

2. Simplify the expression, if possible.

a. $81^{\frac{3}{4}}$ b. $-81^{\frac{3}{4}}$ c. $(-81)^{\frac{3}{4}}$

d. $81^{-\frac{3}{4}}$ e. $-81^{-\frac{3}{4}}$ f. $(-81)^{-\frac{3}{4}}$

3. Convert each expression to radical notation.

a. $8y^{\frac{2}{5}}$ b. $(8y)^{\frac{2}{5}}$ c. $4x^{-\frac{5}{6}}$

d. $(4x)^{-\frac{5}{6}}$ e. $-4x^{-\frac{5}{6}}$ f. $(-4x)^{-\frac{5}{6}}$

4. Simplify the expression by using the properties of rational exponents. Write the final answer using positive exponents only.

a. $\dfrac{-7a^{-\frac{2}{5}}}{a^{\frac{3}{4}}}$ b. $(4x^{\frac{4}{7}}y^{-\frac{1}{3}})^{\frac{7}{2}}$

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c. $\left(\dfrac{-27b^{4}c^{-5}}{b^{-2}c}\right)^{\frac{2}{3}}$

d. $(4x^{\frac{4}{7}}y^{-\frac{1}{3}})^{\frac{7}{2}}$

4. Simplify the radicals. Assume all variables are positive.

a. $\sqrt{54x^{5}yz^{6}}$

b. $-3a\sqrt{27a^{4}b^{7}c^{2}}$

c. $\sqrt{\dfrac{3x^{4}y^{5}}{300xy^{3}}}$

d. $5t\sqrt[3]{75r^{8}st^{6}}$

Radicals, specifically square roots, date back as far as c.1650 B.C., from the time of Egypt's Middle Kingdom. The Rhind Papyrus makes references to square roots since they are tied to the diagonals of squares and rectangles; often applicable in the construction of a temple. The "Rhind Papyrus" is named after Henry Rhind, a Scottish lawyer, who purchased it in Egypt in 1858. It was placed in the British Museum in London, England, in 1864 and is still there today; a fragment is also in the collection of the Brooklyn Museum on Eastern Parkway. The Rhind Papyrus, according to the British Museum website, is a "list of practical problems encountered in administrative and building works. The text contains 84 problems concerned with numerical operations, practical problem-solving, and geometrical shapes."

References:

Robins, G. & Shute, C. (1990). *The Rhind Mathematical Papyrus: An ancient Egyptian text.* New York, NY: Dover.
The British Museum; downloaded on 8/1/ 2011, from
http://www.britishmuseum.org/explore/highlights/highlight_objects/aes/r/rhind_mathematical_papyrus.aspx

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Name: ___________________________________ Points: ______

1. Perform the indicated operation.

a. $3\sqrt{98x^2}+5x\sqrt{18}-6\sqrt{4}$

b. $\left(3\sqrt{6}\right)\left(-2\sqrt{6}\right)$

c. $\dfrac{11}{4\sqrt{3}-2}$

d. $\dfrac{1-\sqrt{3}}{3\sqrt{7}+\sqrt{2}}$

2. Solve the equations containing radicals.

a. $\sqrt{2x+3}=x$

The Origin of the Radical Sign $\sqrt{}$

Before the turn of the 13th century, the root of a number was not symbolized but simply written out using a translation of the word "root" or "side." The word *Radix,* meaning "scale" in Latin, was used in medieval times (the 13th century) in Europe to signify dispensers of medicinal compounds; *Radix* was abbreviated as R_x or ℞, a sign we use today for prescriptions.

The radical sign used in mathematics today was introduced in 1525 by Christoff Rudolf, who was born in Silesia, an area that is now in Poland. He studied at the University of Vienna, and wrote a book on Algebra, entitled *Die Coss*, using German, a language considered "vulgar" because it was spoken by Germanic people. At this time, all "learned" books were written in Latin, considered the language of learning (Eves, 1990).

The radical sign resembles the small *r* for radix (Smith, 1958). French, British, and Italian mathematicians did not immediately accept the symbol. However, the publication of Rene Descartes' book, *La Geometrie*, in 1637, used Rudolf's symbol for root. Descartes' influence helped standardize $\sqrt{}$ in the mathematical world.

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b. $\sqrt{4x-2}=\sqrt{3x+7}$

c. Solve $5+\sqrt{3x-11}=x$

d. Solve $\sqrt{2x+6}-\sqrt{x+4}=1$

References

Smith, D.E. (1958). *History of mathematics*. Vol. 2. Toronto, Canada: General Publishing Company.

Eves, H.W. (1990). *An Introduction to the history of mathematics with cultural connections,* Sixth Edition. Philadelphia, PA: Saunders College Publishing.

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Name: ______________________________ Points: ______

1. Definition of i: $i =$ __________

2. Definition of $\sqrt{-b}$ for $b > 0$ $\sqrt{-b} =$ ______________

3. Simplify the expressions.
   a. $\sqrt{-81}$ b. $\sqrt{-75}$ c. $-\sqrt{-49}$ d. $\sqrt{-15}$

4. Simplify the product or quotient in terms of i
   a. $\dfrac{\sqrt{-36}}{\sqrt{9}}$ b. $\sqrt{-9} \cdot \sqrt{-49}$ c. $\sqrt{-7} \cdot \sqrt{-7}$

5. A **complex number** is a number of the form ______________________ where *a* and *b* are real numbers.

6. The complex number $a + bi$ and ________________ are called **conjugates.**

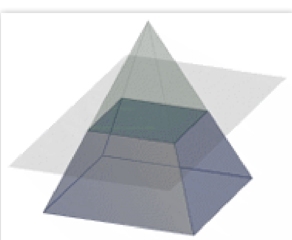

Figure 1

Complex number or imaginary number concept was first investigated by a mathematician and inventor named Heron (c. 10-70 A.D.) from the city of Alexandria on the coast of the Mediterranean, in Egypt. While trying to find the volume of the frustum of a pyramid (see Figure 1) with a square base of a certain size, Heron of Alexandria first encountered the square root of a negative number (Nahin, 1998).

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7. Perform the indicated operation.

a. $\left(\frac{3}{5}+\frac{2}{3}i\right)+\left(\frac{1}{4}-\frac{1}{3}i\right)$

b. $(-5+9i)-(-2+3i)$

c. $4i\left(6-\frac{11}{16}i\right)$

d. $(2+3i)(2-3i)$

e. $\frac{20i}{-2-i}$

f. $\frac{3-4i}{5-3i}$

Reference:
Nahin, J. P. (1998). *An imaginary tale: The story of i.* Princeton, NJ: Princeton University Press.

Name: ___________________________________ Points: ______

1. The **standard form of a quadratic equation** is ___________________________________

2. List the possible methods that a quadratic equation can be solved:

____

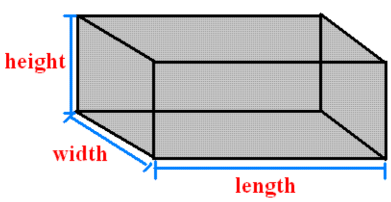
____

____

____

3. A corner shelf is to be made from a triangular piece of marble. Find the distance x that the shelf will extend along the wall. Assume the walls are at right angles. Round the answer to the nearest inches.

4. The volume of a rectangular box is 64 cm^3. If the width is 9 times longer than the height and the length is 3 times longer than the height, find the dimensions of the box. [$volume$ = ($length$)($width$)($height$)]



Schwartzman (1994) defines **Quadratic:** "From the Latin 'quadratum ', 'square', from the Indo-European root k^{w}etwer – 'four' to ancient Romans. The name square was literally a description of the figure as 'four sided.' The Romans following the Greek model, conceived of the abstract quantity s^2 as the area of a square sides, that's why something raised to the second power is said to be squared using the English word quadratic."

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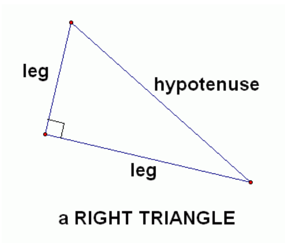
5. The length of one leg of a right triangle is 1 meter more than twice the length of the other leg. The hypotenuse measures 8 meters. Find the lengths of the legs. Round to the nearest tenth.



6. Solve the following equations by completing the square:

a. $2b^2 - 12b = 5$

b. $6k^2 + 17k + 5 = 0$

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Practical Use of the Quadratic Equation

Where there is a problem with unknown values, a formula is set up to help find the unknown values. Such problems can often be found in word problems. The setting up of a formula is called a "quadratic equation." It is believed that the Babylonian civilization of the second millennium B.C. (McLiesh, 1991) was the first to use worded quadratic problems, undoubtedly to solve practical problems.

Many of the principles used in solving word problems were introduced by practitioners of the law, those we would call lawyers today, who defended the interests of their kin against claims by other claimants (Cooke, 1997). To explain how they derived their claims on behalf of their clients, mathematical formulas were used, and these were written out, without the use of mathematical symbols.

Among the early scholars of a center of learning in the City of Baghdad, around 800 A.D., known as the House of Wisdom, was a mathematician and astronomer from the territory known today as Uzbekistan. His name was Muhammed ibn Musa Al-Khwarizmi, (Muhammed, Son of Moses, from Khorezm) (Cooke, 1997). Al-Khwarizmi was known for his work in algebra which can be found in his book with the Arabic title "Kitab fi al-jabr wal-muqabala." In the preface of this book, Al-Khwarizimi states that his work is intended to be for practical use, what might today be called applied mathematics.

7. Solve

a. $p(p-6)=-14$

b. $\frac{1}{5}y^2+y+\frac{3}{5}=0$ (Hint: Clear fractions first)

References

Cooke, R. (1997). *The history of mathematics: a brief course.* New York, NY: John Wiley and Sons.

McLiesh, J (1991). *The history of numbers and how they shape lives*. New York, NY: Ballantine Books.

Swartzman, S. (1994). *The words of mathematics: An etymological dictionary of mathematical terms used in English*. USA: The Mathematical Association of America.

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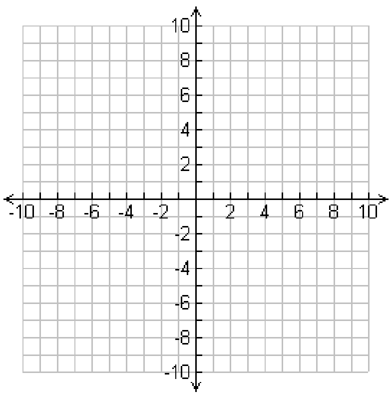
Name: ___________________________________ Points: ______

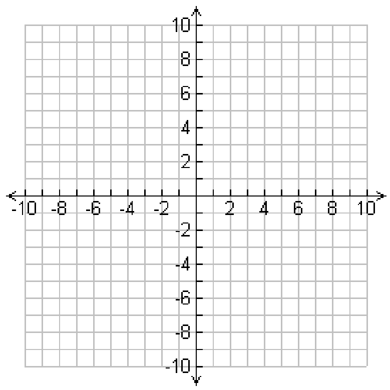
1. For each function:
   i) Identify the vertex.
   ii) Sketch the graph.
   iii) Identify the axis of symmetry.
   iv) Identify the maximum or minimum value of the function.

a. $f(x) = x^2 - 3$



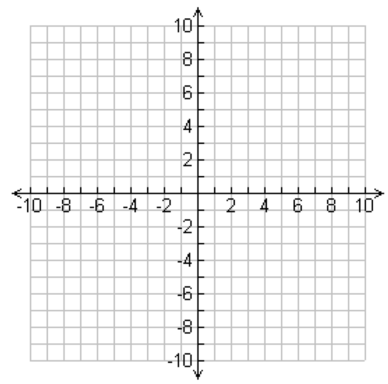
The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .

b. $f(x) = (x-3)^2$



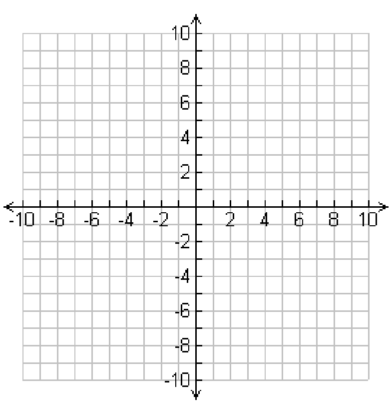
The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .

c. $f(x) = -x^2 - 3$



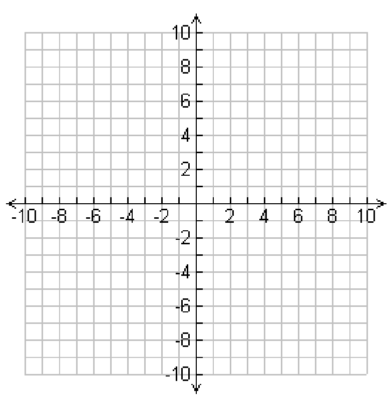
The vertex is __________ .
The axis of symmetry is _________.
The max/min value is __________ .

d. $f(x) = -(x-3)^2$



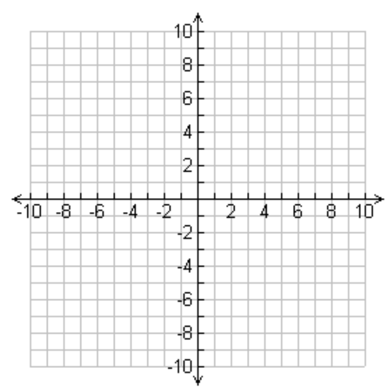
The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .

e. $f(x) = \dfrac{1}{2}(x-3)^2$



The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .

f. $f(x) = 2(x-3)^2$



The vertex is __________ .
The axis of symmetry is _________.
The max/min value is __________ .

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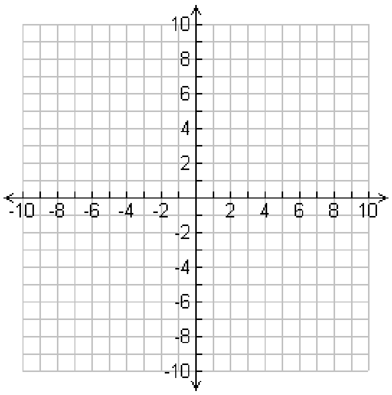
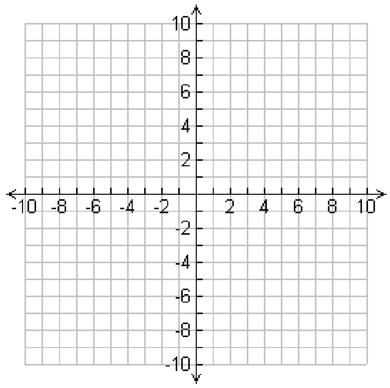
g. $f(x) = (x+2)^2 - 4$



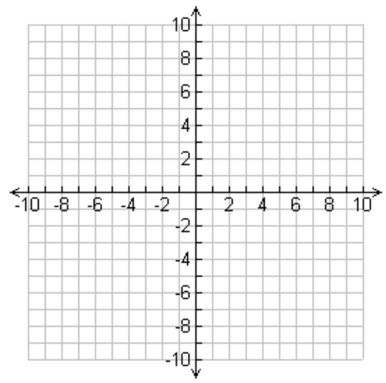
The vertex is ___________ .
The axis of symmetry is __________ .
The max/min value is ___________ .

h. $f(x) = (x-3)^2 - 1$



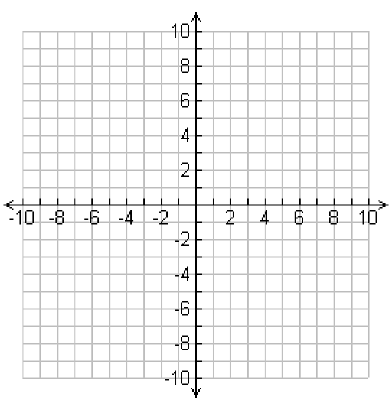
The vertex is ___________ .
The axis of symmetry is __________ .
The max/min value is ___________ .

i. $f(x) = -(x+6)^2 + 9$



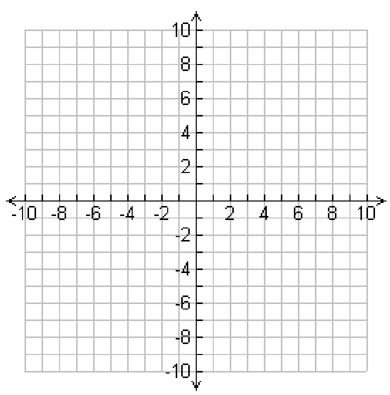
The vertex is ___________ .
The axis of symmetry is __________.
The max/min value is ___________ .

j. $f(x) = -(x-1)^2 + 7$



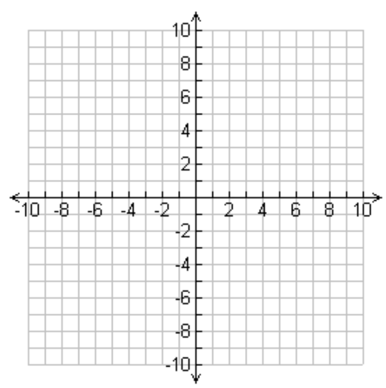
The vertex is ___________ .
The axis of symmetry is __________ .
The max/min value is ___________ .

k. $f(x) = \frac{1}{4}(x-5)^2 + 4$



The vertex is ___________ .
The axis of symmetry is __________ .
The max/min value is ___________ .

l. $f(x) = 3(x-4)^2 - 4$



The vertex is ___________ .
The axis of symmetry is __________.
The max/min value is ___________ .

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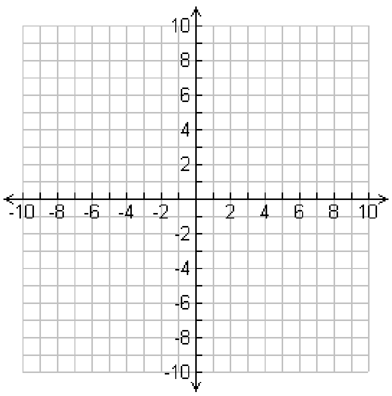
2. Find the vertex by using the vertex formula.

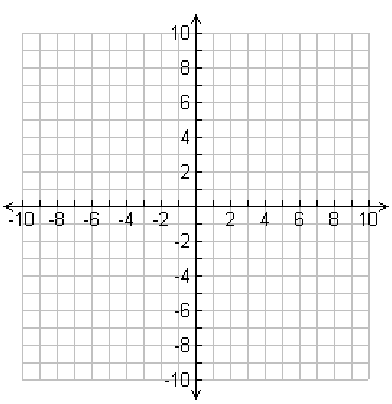
a. $h(x) = x^2 + 6x - 7$

The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .
The x-intercept(s) is, if they exist, __________________ .
The y-intercept is ____________ .



b. $k(x) = 2x^2 + 8x + 9$

The vertex is __________ .
The axis of symmetry is _________ .
The max/min value is __________ .
The x-intercept(s) is, if they exist, __________________ .
The y-intercept is ____________ .



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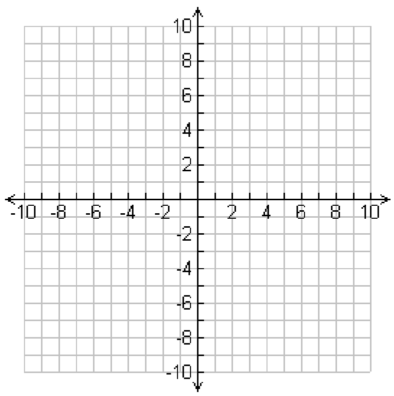
c. $p(x) = -x^2 + 5x - \dfrac{25}{4}$

The vertex is ___________ .
The axis of symmetry is __________ .
The max/min value is ___________ .
The x-intercept(s) is, if they exist, ____________________ .
The y-intercept is _____________ .



Although the Egyptians knew how to calculate the areas of building of various shapes, they were unable to calculate the length of sides or walls for the floor plans. Instead of creating or developing a method in which to calculate the wall dimensions, they developed another method of finding the dimensions by creating a lookup table of standard sizes. This table is similar to that of multiplication tables. Engineers would find the most fitting design based on the table developed. Unfortunately due to incoherent reproduction of the tables, there were instances of errors. Hence this method proved inefficient (Hell, 2004).

Reference

Hell, D. (2004). *History behind Quadratic Equations*. Retrieved on June 22,2011. www.bx.co.uk/dna/h2g2/A2982567 created October 13,2004

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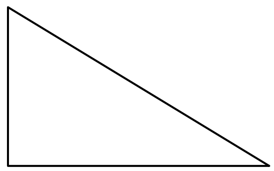
Name: ___________________________________ Points: ______

1.

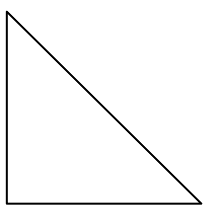


$\sin 30° =$ $\csc 30° =$ $\sin 60° =$ $\csc 60° =$

$\cos 30° =$ $\sec 30° =$ $\cos 60° =$ $\sec 60° =$

$\tan 30° =$ $\cot 30° =$ $\tan 60° =$ $\cot 60° =$

2.

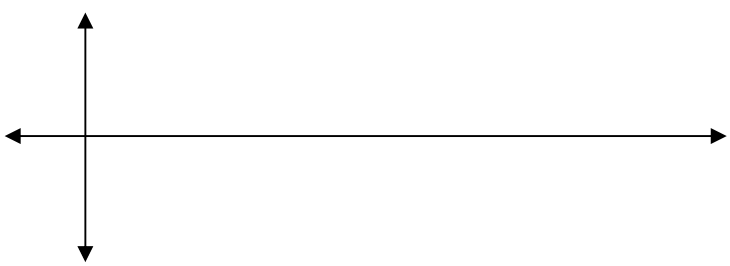


$\sin 45° =$ $\csc 45° =$

$\cos 45° =$ $\sec 45° =$

$\tan 45° =$ $\cot 45° =$

3. Graph of a Sine Function



$\sin 0° =$

$\sin 90° =$

$\sin 180° =$

$\sin 270° =$

$\sin 360° =$

4. Graph of a Cosine Function

$\cos 0° =$

$\cos 90° =$

$\cos 180° =$

$\cos 270° =$

$\cos 360° =$

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5.Change to radians

a. 330°   b. 225°   c. -160°

6. Change to degrees

a. $\frac{\pi}{5}$ b. $\frac{5\pi}{3}$ c. $-\frac{4\pi}{5}$

7. A line passing through point $(2,-3)$. Find the six trigonometric functions.

7. A line passing through point $(-\sqrt{10},-4)$. Find the six trigonometric functions.

Origins of the Coordinate Plane

René Descartes was a noted French Philosopher of the 17th Century who thoughtfully doubted accepted knowledge. It was in Part II of his book "Discours de la methode pour bien conduire sa raison et chercher la verite dans la sciences," published in 1637 (which translates as "Discourse on the method of rightly conducting the reason and seeking truth in science") that he considered how to systematically doubtknowledge. Descartes' re-examination of accepted knowledge was in part a re-examination of geometry. In his search for precision and logic he related components of a geometry figure to two (2) straight lines, what today are called the x and y axes, or the x and y coordinates. This two-dimensional plane is known as the Cartesian plane (Anglin, 1994).

While Descartes' name is associated with the Cartesian plane, the development of a co-ordinate system can be traced back to ancient Egypt, where the idea of a coordinate system was used in the laying out of towns and lands by Egyptian surveyors. From these the Romans acquired the concept of organizing districts. These districts were designated in hieroglyphics by a symbol derived from a grid still used in surveying land today (Smith, 1958).

References

Anglin. W.S. (1994). *Mathematics: A concise history and philosophy.* New York, NY: Springer Verlag.

Smith. D.E. (1958). *History of mathematics, Vol. II.* New York, NY: Dover Publications.

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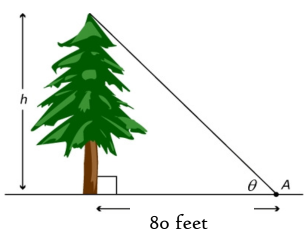
Name: ___________________________________ Points: ______

1. Given $\cos\theta = \frac{1}{5}$ and $\sin\theta < 0$ find the value of the other ratios.

2. Given $\tan\theta = -\frac{7}{3}$ and $\cos\theta < 0$ find the value of the other ratios.

3. Tom wants to measure the height of a tree. He walks exactly 80 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 48º. How tall is the tree?



4. An airplane is flying at a height of 5 miles above the ground. The distance along the ground from the airplane to the airport is 8 miles. What is the angle of depression from the airplane to the airport?

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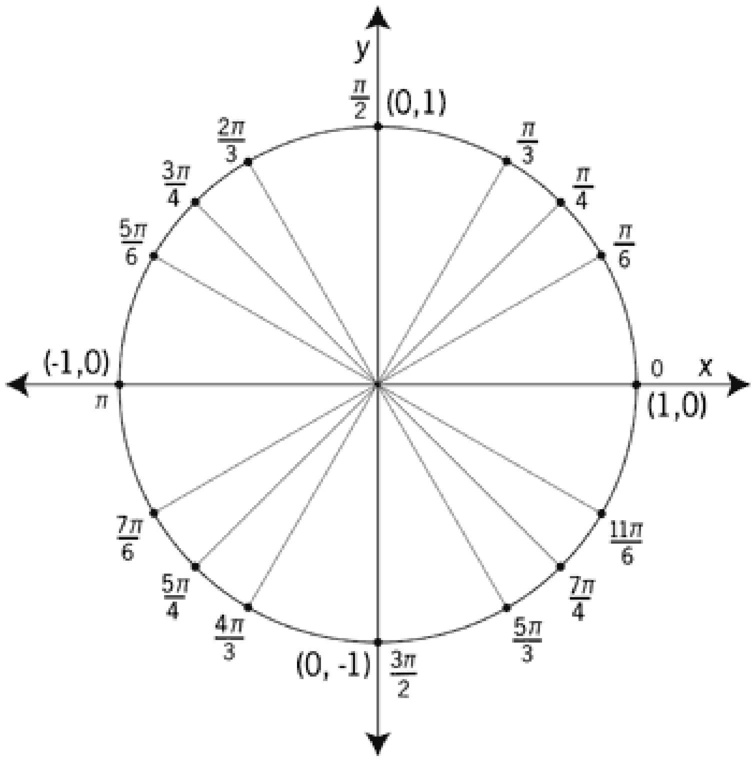
5. Find the coordinates for all the angles in the unit circle:



6. Find the reference angle associated with each rotation and then find the associated point (x, y) on the unit circle.

a. $\theta = \frac{13}{4}\pi$

b. $\theta = \frac{13}{6}\pi$

c. $\theta = -\frac{13}{3}\pi$

d. $\theta = \frac{7}{2}\pi$

The development of Trigonometry spans all cultures. The word is derived from combining two Greek words; *trigonon* which means "triangle" and *metron* "to measure." Around three millennia ago, the Babylonian number system of base sixty (sexagesimal) promoted the idea of 360 degrees in a circle, 60 minutes in a degree, and sixty seconds in a minute. This concept led to having sixty minutes in an hour.

Reference:

Willers, M. (2009). *Algebra: The x and y of everyday math.* New York, NY: Fall River Press.

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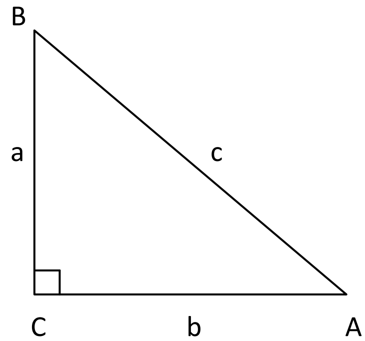
Name: ___________________________________ Points: ______

1. $30°, 45°, 60°$ Trigonometric Functions



Solve each of the right triangles expressing lengths of sides to the nearest unit and angles to the nearest degree.

a) $A = 37°$ and $b = 14$

b) $B = 23°$ and $b = 12$

c) $a = 5$ and $b = 12$

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2. A 30-foot ladder, leaning against the side of a building makes a $50°$ angle with the ground. How far up on the building does the top of the ladder reach? Express your answer to the nearest tenth of a foot.

3. Bill is standing on top of a 175-foot cliff overlooking a lake. The measurement of the angle of depression to a boat on the lake is $29°$. How far is the boat from the base of the cliff? Express your answer to the nearest foot.

It turns out that the "Pythagorean Theorem" about the relationship of squares on the sides of right angles was not invented only by Pythagoras, a Greek mathematician living in ca. 585 - ca. 500 (Eves, 1990). Pythagoras, however, may have been the first to construct a proof of the theorem. This idea, that the sum of squares of the two shorter sides equals the square of the hypotenuse was widely used in India around 300 to 600 B.C. to provide instructions on how to build altars for everyday worship in one's home. Every male head of family was responsible for building an appropriate altar, and the instructions were provided in a "Sulva Sutra."

Reference:
Eves, H.W.(1990). *An Introduction to the history of mathematics: a brief course.* New York, NY: John Wiley and Sons.

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Name: ___________________________________ Points: ______

1. Prove the identity: $1+\cos A = \dfrac{\sin^2 A}{1-\cos A}$

2. Prove the identity: $\tan B + \cot B = \sec B \csc B$

3. Prove the identity: $\sin x \cot^2 x + \sin x = \csc x$

4. Prove the identity: $\sin^2 a = \dfrac{1-\cos^4 a}{1+\cos^2 a}$

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5. Prove the identity: $\dfrac{1+\tan y}{1+\cot y}=\tan y$

6. Prove the identity: $\dfrac{1}{\cos B}-\cos B=\sin B\tan B$

7. Prove the identity: $2\csc^2 t=\dfrac{1}{1-\cos t}+\dfrac{1}{1+\cos t}$

During the years of 162-127 BC lived Hipparchus in the town of Nicaea, near present day Istanbul. Theon, a Greek scholar and mathematician who lived in Alexandria, Egypt, dubbed Hipparchus "The Father of Trigonometry" (Cajori, 1985). This honorific name was given to Hipparchus because he undertook the task of calculating and tabulating corresponding values of arc and chord for a whole series of angles (Boye, 1968). Hipparchus evidently drew up the tables for his research in astronomy; mathematics was a means to study the angles and movement of the stars and planets, his real interest.

References:
Boye, C.B. (1968). *A history of mathematics (2^{nd} Ed.).* New York, NY: John Wiley and Sons
Cajori, F. (1985). *A history of mathematics.* New York, NY: Chelsea Publishing Company.

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Name: ________________________________ Points: ______

1. Solve for x: $0 \le x < 360°$

a) $3\sin^2 x = \sin x$

b) $2\cos^2\theta - \cos\theta = 1$

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c) $\tan^3 A = \sqrt{3}\tan x$

d) $3\sin^2 x - 5\cos x = 1$

Table of Values for Trigonometric Functions

An Indian mathematician known by the name of Aryabhata (476-550 AD) developed the ratios for sine and cosine. Bhaskara, an Indian mathematician in the seventh century AD, found a fairly precise formula to calculate the sine of *x* using radians and not degrees:

$\sin x = \dfrac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ for $0 \le x \le \dfrac{\pi}{2}$. Then in the ninth century, Al-Khwarizmi was able to create a table for sine, cosine, and tangent. After a century, Islamic mathematicians had access to all six ratios and had tables accurate to eight decimals.

Reference:
Willers, M. (2009). *Algebra: The x and y of everyday math.* New York, NY: Fall River Press.

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