MODULE 1	ABSOLUTE VALUE INEQUALITIES, LINES, AND FUNCTIONS

Name:_____ Points:_____

Exercise 1. Solve for x. Write your answer in interval notation.

(a) $\qquad 2 \cdot \left|4x - 12\right| < 8$

(b) $\qquad (-2) \cdot \left|4x - 12\right| \leq -8$
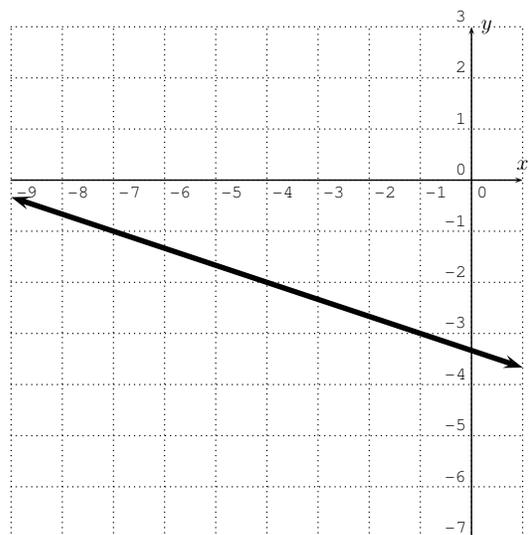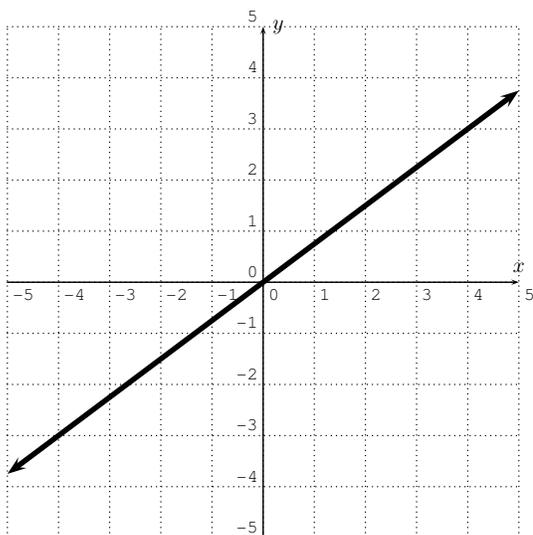
(c) $\left|7x+5\right| > 3$

(d) $\left|x+4\right| < -2$

(e) $\left|x+4\right| > -2$

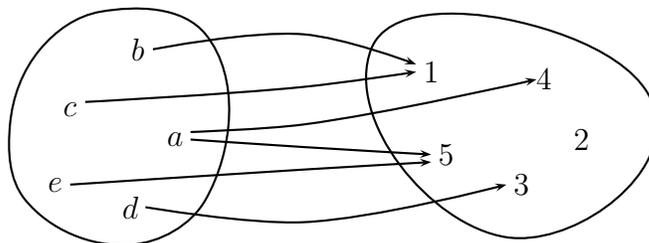**Exercise 2.** Find the equation of the line in slope-intercept form.

(a)

(b)

Exercise 3. Determine if the following assignments are functions. Justify your answer.

(a)

x	3	4	7	4
y	7	4	7	4

(b)

Can you define a function with domain and range given below? Justify your answer.

(c)   domain = set of all college students in the U.S.
         range = set of all colleges in the U.S.

(d)   domain = set of all colleges in the U.S.
         range = set of all college students in the U.S.

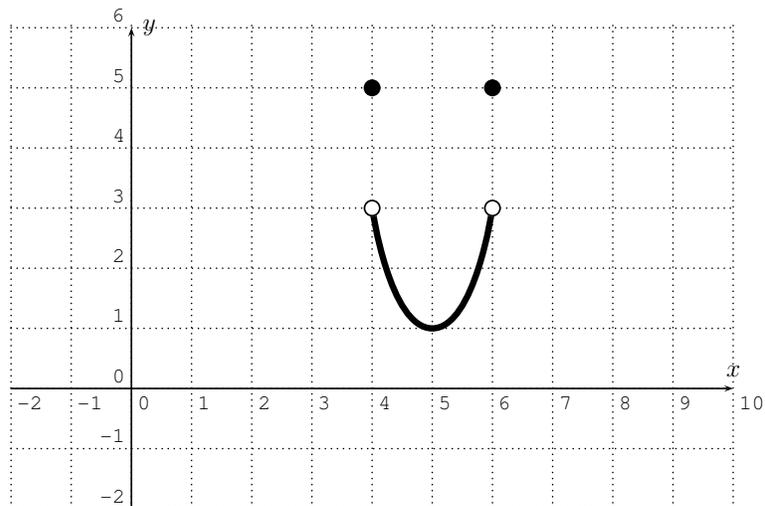| MODULE 2	FORMULAS AND GRAPHS, ROOTS, MAXIMA AND MINIMA

Name:_____ Points:_____

Exercise 1.

(a)   Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = 3x^2 + 2x - 1$.

(b)   Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = x^3$.

(c)   Find the difference quotient $\frac{f(x)-f(a)}{x-a}$ for $f(x) = x^2$.

Exercise 2. Consider the graph of a function $y = f(x)$ displayed below.

Find the following data.

(a)   Domain of $f =$

(b)   Range of $f =$

(c) $f(5) =$

(d) $f(6) =$

(e) $f(7) =$

(f) $f(4.5) =$

Exercise 3.

(a)   Find all roots of $f(x) = x^3 - 3x - 1$ and approximate them to the nearest hundredth.

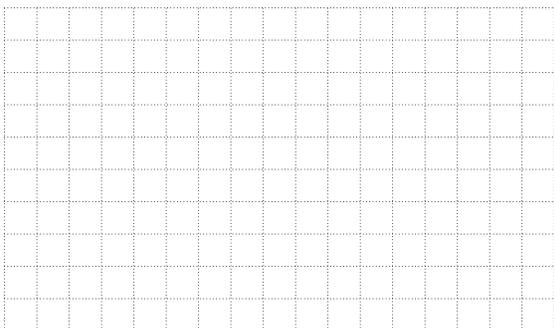(b)   Find all maxima and minima of $f(x) = x^4 - 5x^2 + 4$ and approximate them to the nearest thousandth.

(c)   Find all maxima and minima of $f(x) = x^3 - 12x^2 - 100x + 1200$ and approximate them to the nearest tenth.

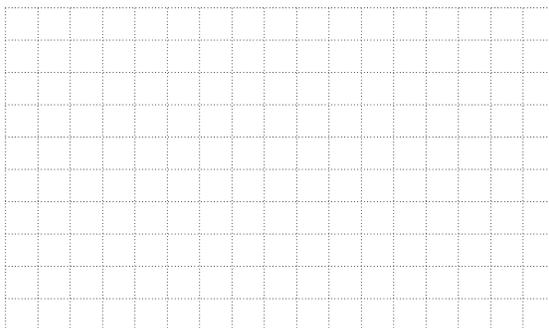| MODULE 3	TRANSFORMATIONS OF GRAPHS AND OPERATIONS ON FUNCTIONS

Name:_____ Points:_____

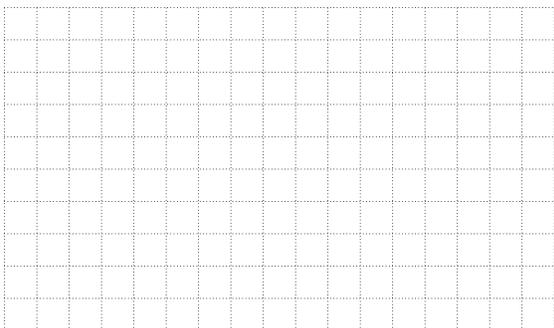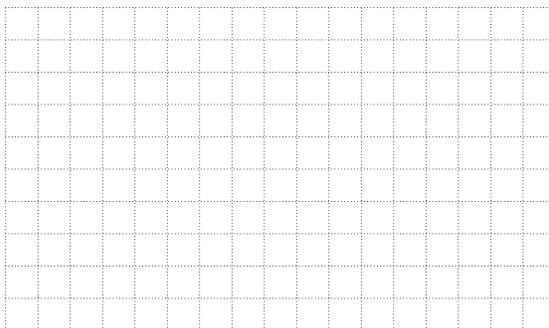**Exercise 1.** Sketch the graph of the function. Check your answer with the calculator.

a) $y = (x+4)^2 - 3$

b) $y = -\dfrac{1}{x-3}$

c) $y = \sqrt{-(x+2)}$
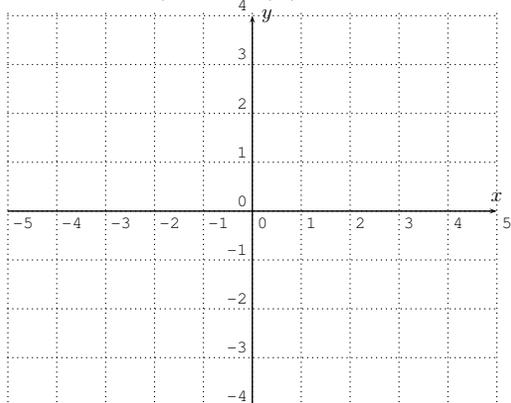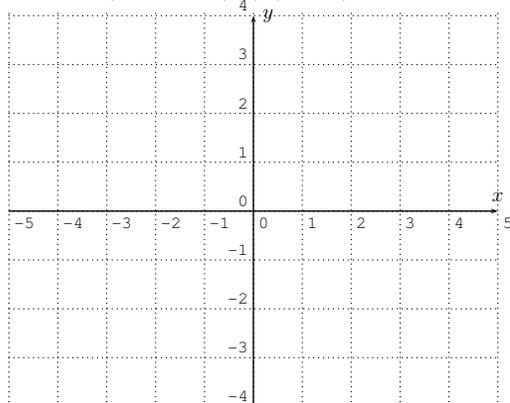
d) $y = (x-4)^3$

Exercise 2. The graph of the function $y = f(x)$ is displayed below.

1
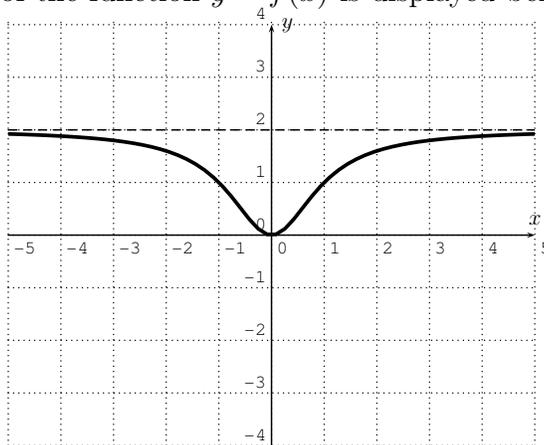
Sketch the graphs of the transformed functions below.

a) $y = f(x) + 2$

b) $y = -(f(x) + 2)$

c) $y = -f(x)$
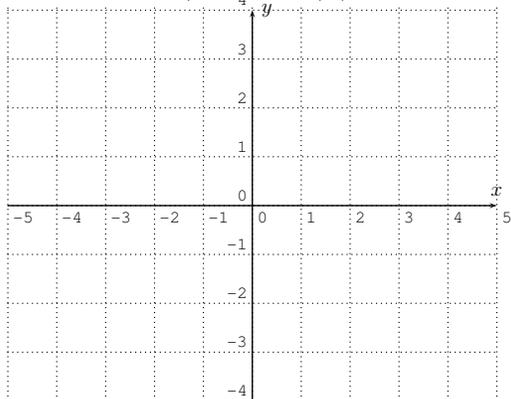
d) $y = -f(x) + 2$
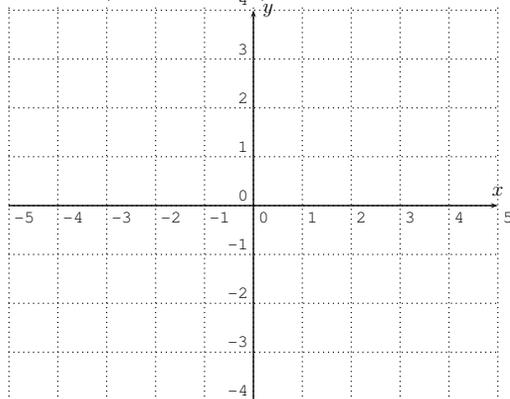
Exercise 3. Let $f(x) = 3x + 2$ and $g(x) = x^2 - 7x + 4$. Find the following compositions.

(a) $(f \circ g)(x) =$

(b) $(g \circ f)(x) =$

Now, let $f(x) = \frac{1}{x+2}$, $g(x) = \sqrt{x+3}$, and $h(x) = x^3 + 4$. Find the compositions:

(c) $(f \circ g \circ h)(x) =$

(d) $(h \circ f \circ g)(x) =$

Exercise 4. Complete the table by calculating the compositions.

x	2	4	6	8	10
$f(x)$	6	8	4	4	2
$g(x)$	2	7	8	2	4
$(f \circ g)(x)$					
$(g \circ f)(x)$					
$(g \circ g)(x)$					

MODULE 4	INVERSE FUNCTIONS AND LONG DIVISION

Name:_____ Points:_____

Exercise 1. Find the inverse of the given function.

(a) $f(x) = \dfrac{1}{x-5} + 5$

(b) $f(x) = \dfrac{3x^2 - 7}{8 - 5x^2}, \quad \text{for } x \geq 0$

(c) $f(x) = x^2 + 3$,   for $x \geq 0$

(d) $f(x) = x^2 + 3$,   for $x \leq 0$

Exercise 2. Check if the functions are inverses of each other. If so, what are the domains and ranges where they are inverses?

(a) $f(x) = \sqrt{x + 6}$ and $g(x) = (x - 6)^2$

(b) $f(x) = |x|$ and $g(x) = x$

(c) $f(x) = |x|$ and $g(x) = -x$

Exercise 3. Divide using long division.

(a) $(x^4 + 3x^3 - 2x^2 + 9x + 8) \div (x + 4) =$

(b) $(6x^3 + 5x^2 - 14x - 10) \div (2x + 3) =$

Exercise 4.

(a) Check that 2 is a root of $f(x) = x^5 - 4x^3 + 7x - 14$ and use this to factor f.

(b) Check that -3 is a root of $f(x) = x^3 + 8x^2 + 18x + 9$ and use this to factor f.

MODULE 5	ROOTS AND GRAPHS OF POLYNOMIALS

Name:_____   Points:_____

Exercise 1. Multiply and write your answer as a polynomial in descending degree (that is in the form $ax^2 + bx + c$).

(a)   Multiply $(x - (3 + 2i)) \cdot (x - (3 - 2i)) =$

(b)   Multiply $(x - 5) \cdot (x - (4 + 6i)) =$

Note: The above examples confirm again that a polynomial has real coefficients exactly when for each complex root $c = a + bi$ its complex conjugate $\bar{c} = a - bi$ is also a root.

Exercise 2.

(a)   Find a polynomial of degree 4 whose roots include 2, -3, and so that $f(0) = 10$.

(b)   The following graph is the graph of a polynomial of degree 5 which displays all of the roots of the polynomial. What is a possible formula for the polynomial?
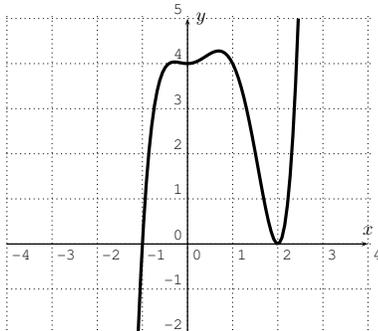
Exercise 3. Let $f(x) = x^3 - x^2 - 10x + 12$.

(a)   Find all roots of the polynomial **without** approximation. Write your answer in
           simplest radical form.

(b)   Sketch a complete graph of the function f. Include all roots, all maxima, and
           all minima.
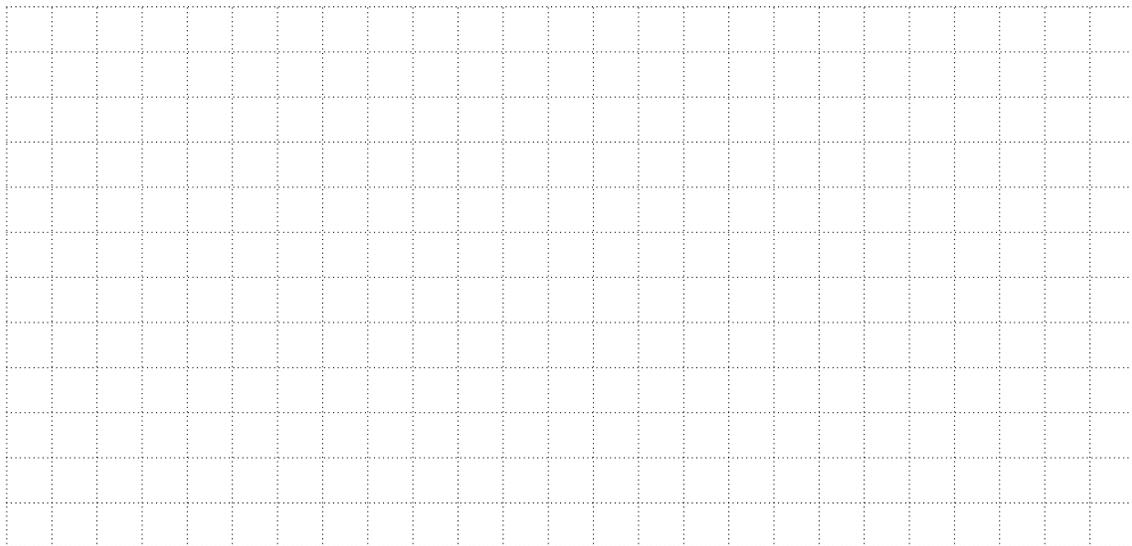
Exercise 4. Factor **completely**.

(a) $\qquad y = x^4 + 2x^3 - 3x^2 - 8x - 4$

(b) $\qquad y = x^6 + 2x^5 + x^4 + 2x^3$

| MODULE 6 | RATIONAL FUNCTIONS AND INEQUALITIES |

Name:_____ Points:_____

Exercise 1. Find the domain, vertical asymptotes, removable singularities, horizontal asymptotes, and x- and y-asymptotes. Sketch the graph.

(a) $\qquad f(x) = \frac{6-x}{x^2-6x+8}$
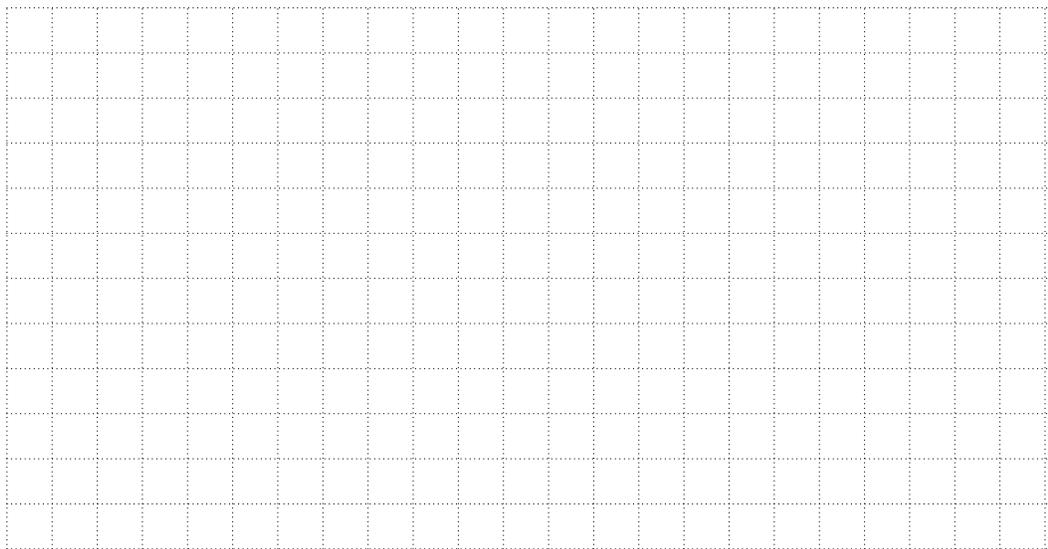
(b) $f(x) = \frac{x^2-9}{x^2+6x+5}$
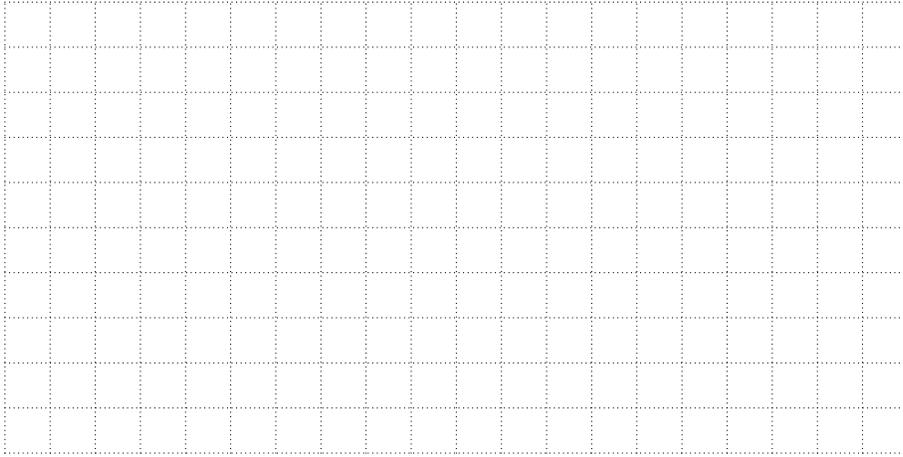
(c) $f(x) = \frac{4-x^2}{x-1}$
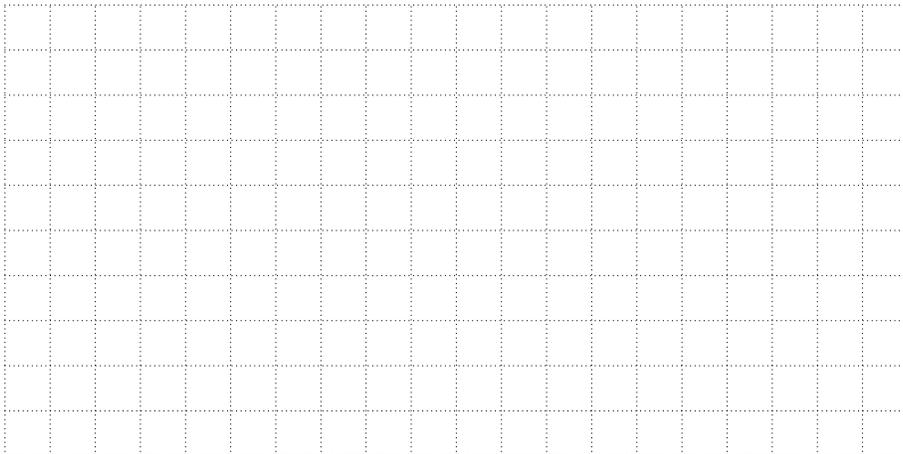
(d) $\qquad f(x) = \frac{x^2 - 2x - 3}{x^2 - 1}$
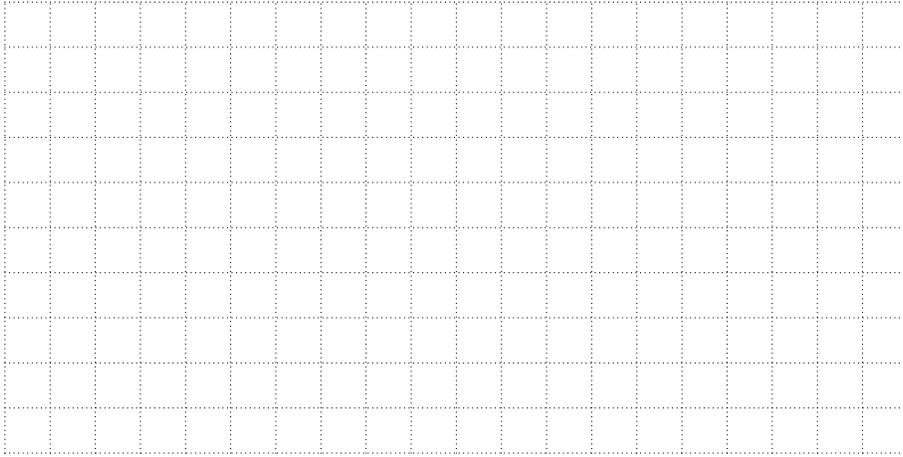
Exercise 2. Solve for x.

(a) $\qquad x^2 - 5x + 5 > 0$

(b) $\frac{x+7}{x^2-4} \geq 0$

(c) $|2x + 3| < 7$

(b) $\frac{x+7}{x^2-4} \geq 0$

MODULE 7	**EXPONENTIAL AND** **LOGARITHMIC FUNCTIONS**

Name:_____   Points:_____

Exercise 1. Evaluate the logarithms.

(a) $\log_6(36) =$

(b) $\log_{0.2}(125) =$

(c) $\log_4(8) =$

(d) $\log_7(14) =$

Exercise 2. Find the domain of the given function.

(a) $f(x) = \log_2(8 - 6x)$

(b) $f(x) = \ln(x^2 - 4)$

(c) $f(x) = \log(x)$ Sketch the graph of $f(x) = \log(x)$:
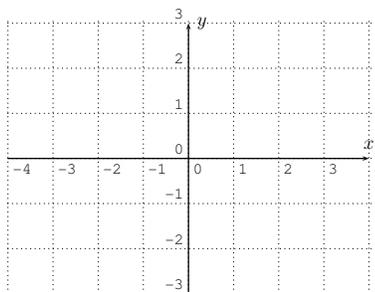
(d) $f(x) = \sqrt{\log(x)}$

(e) $f(x) = \frac{1}{\log(x)}$

Exercise 3. Assume $x, y, z > 0$.

(a)   Combine to one logarithm:

$\frac{1}{2}\log_5(x) - 3\log_5(y) - \log_5(z) =$

(b)   Expand in terms of $u = \log_2(x), v = \log_2(y), w = \log_2(z)$:

$\log_2\left(\frac{z^2}{\sqrt{x \cdot y}}\right) =$

(c)   Combine to one logarithm: (Hint:use the change of base formula!)

$\log_2(x) + \log_3(y) =$

Exercise 4. Solve for x:

(a) $\qquad 3^{x+5} = 9^{x+1}$

(b) $\qquad 1.03^x = 6$

(c) $\qquad 20 \cdot 1.2^x = 37$

(d) $\qquad \log_3(x-2) + \log_3(x+4) = 3$

MODULE 8	APPLICATIONS OF EXPONENTIALS AND LOGARITHMIC FUNCTIONS

Name:_____ Points:_____

Exercise 1. Solve for x.

(a) $\qquad 2.7^{x+2} = 6.5^x$

(b) $\qquad 5^{x+3} = 9^{x+1}$

Exercise 2. A bacterial culture of 20g has been cultivated, which naturally increases at a rate of 3.5% per week.

(a)   What will be the weight of the culture after 6 weeks?

(b)   How long will it take until the culture has doubled in weight?

Exercise 3. A radioactive substance decays with a half-life of 4 hours. How long will it take until 34mg will have decayed to 10mg?

Exercise 4. A piece of wood has lost 12% of its carbon-14. How old is the wood?

Exercise 5. $5,000 have been invested for 10 years at a rate of 4.2% with a monthly compounding. How much money will the investor receive at the end of the investment period?

$\boxed{\textbf{MODULE 9}}$ $\boxed{\textbf{THE TRIGONOMETRIC FUNCTIONS}}$

Name:＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿ Points:＿＿＿＿＿＿＿

Exercise 1. Find the trigonometric function values.

(a) $\quad \sin(120°) =$

(b) $\quad \cos\left(-\frac{7\pi}{4}\right) =$

(c) $\quad \tan\left(\frac{5\pi}{3}\right) =$

(d)   Assume that $\sin(\alpha) = \frac{3}{5}$, $\cos(\alpha) = \frac{4}{5}$. Find $\tan(\alpha) =$

(e)   Assume that $\cos(\beta) = -\frac{5}{13}$, and that β is in quadrant III. Find $\sin(\beta) =$

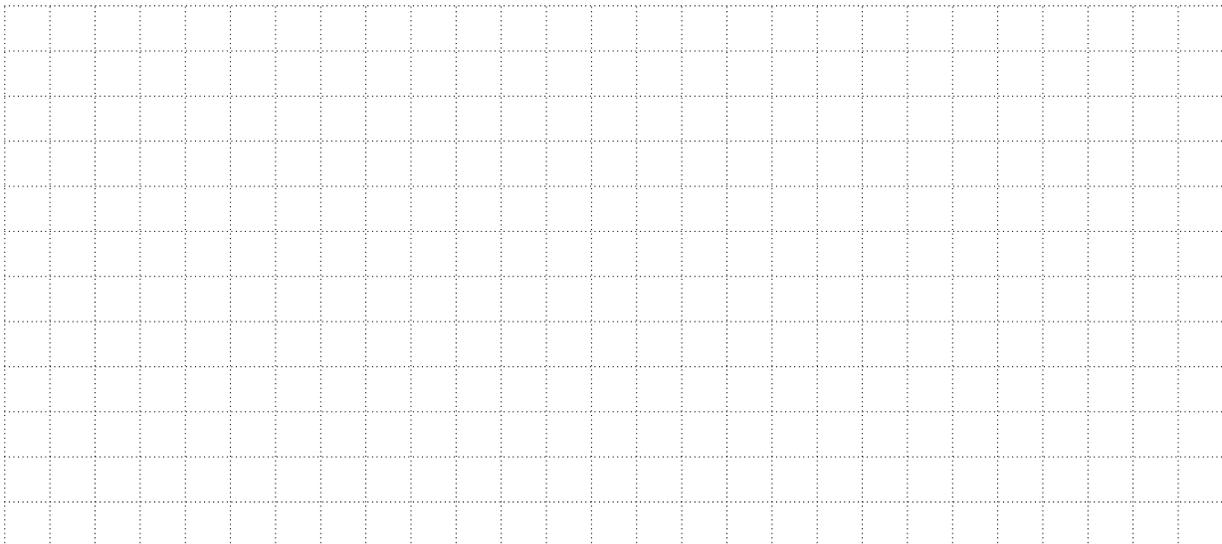**Exercise 2.** Find the amplitude, period, and phase shift. Graph the function over one full period. Label all maxima, minima, and x-intercepts.

(a) $f(x) = 4\sin(2x - \pi)$

          amplitude $=$

          period$=$

          phase shift$=$

(b) $f(x) = 5\cos(4x + 3\pi)$

amplitude $=$

period$=$
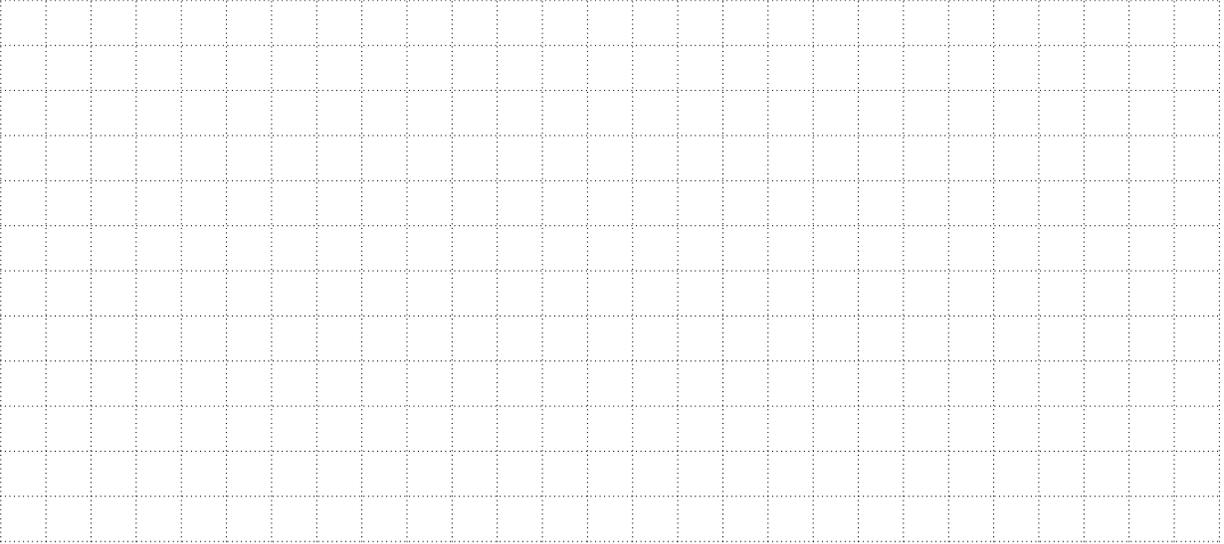
phase shift$=$

(c) $f(x) = 7\sin(6x + \frac{\pi}{3})$

amplitude $=$

period$=$

phase shift$=$
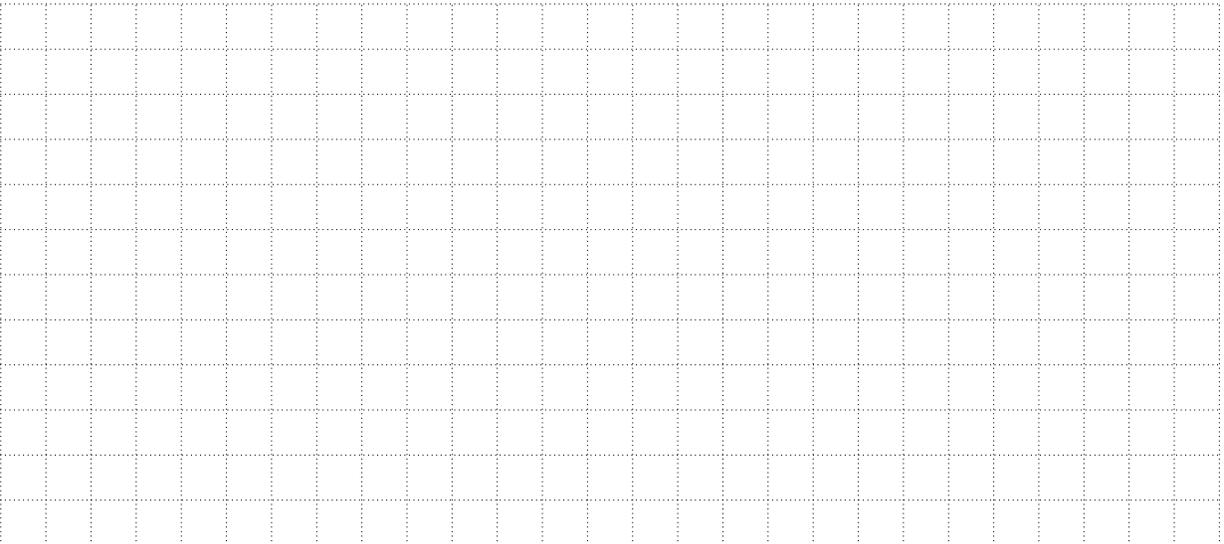
$$\boxed{\textbf{MODULE 10}} \quad \boxed{\begin{array}{l} \textbf{SOLVING} \\ \textbf{TRIGONOMETRIC EQUATIONS} \end{array}}$$

Name:_____ Points:_____

Exercise 1. Solve for x.

(a) $\quad \tan(x) = -\sqrt{3}$

(b) $\quad \sin(x) = -\frac{\sqrt{3}}{2}$

(c) $\quad \csc(x) = 2$

(d) $2\cos(x) + 1 = 0$

(e) $\cos^2(x) - 1 = 0$

(f) $\sin^2(x) + 3\sin(x) + 2 = 0$

(g) $4\cos^2(x) - 3 = 0$

(h) $2\sin^2(x) + 7\sin(x) + 3 = 0$

(i) $3\tan^2(x) - 4\sqrt{3}\tan(x) + 3 = 0$

MODULE 11	COMPLEX NUMBERS AND VECTORS

Name:_____ Points:_____

Exercise 1. Find the **absolute value** and the **angle** of the complex number below.

(a) $-3 + 4i$

(b) $6 - 6i$

Exercise 2. Perform the operation and write your answer in standard $a + bi$ form.

(a) $\dfrac{4\left(\cos \frac{10\pi}{21} + i\sin \frac{10\pi}{21}\right)}{6\left(\cos \frac{\pi}{7} + i\sin \frac{\pi}{7}\right)} =$

(b) $4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right) \cdot 9\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right) =$

(c) $\dfrac{21\left(\cos(165°)+i\sin(165°)\right)}{15\left(\cos(195°)+i\sin(195°)\right)} =$

(d) $\left[5\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^2 =$

(e) $\left[5\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^3 =$

The above generalizes to the so called **De Moivre formula**:

$$\boxed{\left[r\left(\cos(\theta) + i\sin(\theta)\right)\right]^n = r^n\left(\cos(n \cdot \theta) + i\sin(n \cdot \theta)\right)}$$

Exercise 3. Find the **magnitude** and the **direction angle** of the vector below.

(a) $\vec{v} = \langle -2, -2\sqrt{3} \rangle$

(b) $\vec{v} = \langle \sqrt{15}, -\sqrt{5} \rangle$

Exercise 4. Perform the operation for $\vec{v} = \langle -4, 6 \rangle$ and $\vec{w} = \langle -1, -3 \rangle$.

(a) $6\vec{v} - 4\vec{w} =$

(b) $\vec{w} + 7\vec{i} + 8\vec{j} =$

Name:_____ Points:_____

Exercise 1. Find the sum.

(a) $\displaystyle\sum_{k=1}^{5}(k^2 + 2k) =$

(b)   For the sequence $a_1, a_2, a_3, \ldots$ given by $3, 1, 2, -1, 3, -4, 7, -11, \ldots$ find
$$\sum_{\ell=1}^{9} a_\ell =$$

(c)   For the sequence $a_1, a_2, a_3, \ldots$ given by $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ find
$$\sum_{n=3}^{6} a_n =$$

(d)   For the arithmetic sequence given by $7, 16, 25, 34, \ldots$ find
$$\sum_{j=1}^{450} a_j =$$

(e)   For the geometric sequence given by $6, 12, 24, 48, \ldots$ find
$$\sum_{i=1}^{15} a_i =$$

(f)   For the arithmetic sequence given by $-13, -16, -19, -22, \ldots$ find
$$\sum_{k=1}^{2345} a_k =$$

(g)   For the geomteric sequence given by $-4, -2, -1, -\frac{1}{2}, \ldots$ find
$$\sum_{j=1}^{\infty} a_j =$$

(h)   For the arithmetic sequence given by $25, 29, 33, 37, \dots$

find $\displaystyle\sum_{j=1}^{600} a_j =$

find $\displaystyle\sum_{j=1}^{199} a_j =$

find $\displaystyle\sum_{j=200}^{600} a_j =$

(i)   For the geometric sequence given by $-6, 2, -\frac{2}{3}, \frac{2}{9}, \dots$ find
$\displaystyle\sum_{n=1}^{\infty} a_n =$

(j)   For the arithmetic sequence given by $2, 4, 6, 8, 10, \dots$ find
$\displaystyle\sum_{k=1}^{\infty} a_k =$