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TCET 2220
Homework 4

(3-1) A traveling wave of current in milliamperes is given by with t in seconds and x in meters. Determine the following:

$$□ = □\ □□□(□□ * □□^{□}□ - □.□□□□)$$

(a) Direction of propagation

Since the argument of the function has the difference between the time and displacement terms, the wave is traveling in the positive x-direction

(b) Peak value

$$I_{□□} = 8\text{ A}$$

(c) Angular frequency

$$2´ * 10^{□}\text{ rad/sec}$$

(d) Phase constant

$$ϐ = 0.025\text{ rad/m}$$

(e) Cyclic frequency

$$F = \frac{ɣ}{2´} = \frac{2´ * 10^{□}}{2´} = 1\text{MHz}$$

(f) Period

$$T = \frac{1}{F} = \frac{1}{1\text{MHz}} = 1\text{ us}$$

(g) Wavelength

$$ᢆ = \frac{2´}{ϐ} = \frac{2´}{0.025} = 251\text{ meter}$$

(h) Velocity of propagation

$$V = F * ᢆ = 1\text{MHz} * 251.2\text{ m} = 251.2 * 10^{□}\text{ m/s}$$

(3-2) A traveling wave of voltage in volts in given by

$$□ = □□\ □□□(□□^{□}□ + □.□□□)$$

with t in seconds and x in meters. Determine the following:

(a) Direction of propagation

Since the argument of the function has the difference between the time and displacement terms, the wave is traveling in the negative x-direction

(b) **Peak value**

$$V_{□□} = 15\ \text{V}$$

(c) **Angular frequency**

$$10^{□}\ \text{rad/sec}$$

(d) **Phase constant**

$$\beta = 0.35\ \text{rad/m}$$

(e) **Cyclic frequency**

$$F = \frac{\omega}{2\pi} = \frac{10^{□}}{2\pi} = 15.9\ \text{MHz}$$

(f) **Period**

$$T = \frac{1}{F} = \frac{1}{15.9\ \text{MHz}} = 6.3 * 10^{□□}\ □□□$$

(g) **Wavelength**

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.35} = 17.9\ \text{meter}$$

(h) **Velocity of propagation**

$$V = F * \lambda = 15.9\ \text{MHz} * 17.9\ \text{m} = 2.8 * 10^{□}\ \text{m/s}$$

(3-3) A sinusoidal current with a peak value of 2 A and a frequency of 50 MHz is traveling in the positive x-direction with a velocity of $□ * □□^{□}\ □/□$. Determine the following:

(a) **Period**

$$T = \frac{1}{F} = \frac{1}{50\ \text{MHz}} = 2 * 10^{□□}\ □□□$$

(b) **Angular Frequency**

$$2\pi * 50\text{MHz} = 314 * 10^{□}\ \text{rad/se}$$

(c) **Phase constant**

$$\beta = \frac{\omega}{□} = \frac{314 * 10^{□}}{2 * 10^{□}} = 1.57\ □□□/□□□$$

(d) **Wavelength**

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.57} = 4\ \text{m}$$

(e) **An equation for the current**

$$i = 2\cos(314 * 10^{6} t - 1.57 x)$$

(3-4) A sinusoidal voltage with a peak value of 25 V and a radian frequency of 20 Mrad/s is traveling in the negative x-direction with a velocity of $3 * 10^{8}\ m/s$. Determine the following:

(a) Cyclic frequency

$$F = \frac{\omega}{2\pi} = \frac{20\ \text{MHz}}{2\pi} = 3.18\ \text{MHz}$$

(b) Period

$$T = \frac{1}{F} = \frac{1}{3.18\ \text{MHz}} = 3.14 * 10^{-7}$$

(c) Phase constant

$$\beta = \frac{\omega}{v} = \frac{20 * 10^{6}}{3 * 10^{8}} = 0.06\ rad/m$$

(d) Wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.06} = 106.67\ \text{m}$$

(e) An equation for the current

$$V = 25\cos(20 * 10^{6} t + 0.06x)$$

(3-5) Consider the current traveling wave of Problem 3-1. Determine the following:

(a) A fixed phasor representation in peak units as either $\bar{I}^{+}$ or $\bar{I}^{-}$ (You decide which label is appropriate.)

$$\bar{I}^{+} = [I_{pk} * e^{j0}] = 8\angle 0\ A$$

(b) The corresponding distance-varying phasor $\bar{I}(x)$ in peak units

$$\bar{I}(x) = \bar{I}^{+} * [e^{-j\beta x}] = 8e^{j0} * e^{-j0.025x} = 8\angle -0.025x$$

(c) The wave of the distances-varying phasor at x = 100 m.

$$\bar{I}(100) = 8\angle -0.025(100) = 8\angle -2.5$$

(3-6) Consider the voltage traveling wave of Problem 3-2. Determine the following:

(a) A fixed phasor representation in peak units as either $\bar{V}^{+}$ or $\bar{V}^{-}$ (You decide which label is appropriate.)

$$\bar{V}^{+} = [V_{pk} * e^{j0}] = 15\angle 0\ V$$

(b) The corresponding distance-varying phasor $\bar{V}(x)$ in peak units.

$$\bar{V}(x) = 15\angle 0.35x$$

(c) The wave of the distances-varying phasor at x = 4 m

$$\bar{V}(4) = 15\angle 0.35 * 4 = 15\angle 1.40\ V$$

(3-7) Repeat the analysis of Problem 3-5 if the current of Problem 3-1 has a fixed phase shift such that it is described by

$$i = 8\cos(2\pi * 10^{6}t - 0.025x + 1.5)$$

(a) A fixed phasor representation in peak units as either $\bar{I}$ or $\bar{V}$ (You decide which label is appropriate.)

$$\bar{I} = I_{pk} * e^{j\theta} = 8\angle 1.5\ A$$

(b) The corresponding distance-varying phasor $\bar{I}(x)$ in peak units

$$\bar{I}(x) = \bar{I} * e^{-j\beta x} = (8\angle 1.5) * (1\angle -0.025x) = 8\angle 1.5 - 0.025x$$

(c) The wave of the distances-varying phasor at x = 100 m.

$$\bar{I}(100) = 8\angle 1.5 - 0.025(100) = 8\angle -1.0\ A$$

(3-8) Repeat the analysis of Problem 3-6 if the voltage of Problem 3-2 has a fixed phase shift such that it is described by

$$v = 15\cos(10^{9}t + 0.35x + \pi/3)$$

(a) A fixed phasor representation in peak units as either $\bar{I}$ or $\bar{V}$ (You decide which label is appropriate.)

$$\bar{V} = V_{pk} * e^{j\theta} = 15\angle\frac{\pi}{3}\ V$$

(b) The corresponding distance-varying phasor $\bar{V}(x)$ in peak units.

$$\bar{V}(x) = \left(15\angle\frac{\pi}{3}\right) * (1\angle 0.35x) = 15\angle\frac{\pi}{3} + 0.35x$$

(c) The wave of the distances-varying phasor at x = 4 m

$$\bar{V}(4) = 15\angle\frac{\pi}{3} + 1.40\ V$$

(3-9) Redefine the fixed phasor of Problem 3-5 so that the phasor magnitude is expressed in rms units, and determine the average power dissipated in a 50-Ω resistance.

$$|I_{rms}| = |I_{pk}| * 0.707 = 8\ A * 0.707 = 5.66\ A$$

$$P = I_{rms}^{2} * R = (5.66^{2}\ A) * (50\Omega) = 1{,}601.78\ W$$

(3-10) Redefine the fixed phasor of Problem 3-6 so that the phasor magnitude is expressed in rms units, and determine the average power dissipated in a 75-□ resistance.

$$|V_{rms}| = |V_{pk}| * 0.707 = 15\ v * 0.707 = 10.61\ V$$

$$P = \frac{V_{rms}^{2}}{R} = \frac{10.61^{2}\ V}{75\Omega} = 1.50\ W$$

(3-11) Redefine the fixed phasor of Problem 3-7 so that the phasor magnitude is expressed in rms units. Would the power dissipated in a 50-□ resistance be the same as in Problem 3-9?

$$|I_{rms}| = |I_{pk}| * 0.707 = 8\ A * 0.707 = 5.66\ \angle 1.5\ A$$

$$P = I_{rms}^{2} * R = (5.66^{2}\ A) * (50\Omega) = 1{,}601.78\ W$$

Yes they are the same.

(3-12) Redefine the fixed phasor of Problem 3-8 so that the phasor magnitude is expressed in rms units. Would the power dissipated in a 75-□ resistance be the same as in Problem 3-10?

$$|V_{rms}| = |V_{pk}| * 0.707 = 15\ v * 0.707 = 10.61\ V$$

$$P = \frac{V_{rms}^{2}}{R} = \frac{10.61^{2}\ V}{75\Omega} = 1.50\ W$$

Yes they are the same.

(3-13) Under steady-state ac conditions, the forward current wave on a certain lossless 50-□ line is □ = □∠□ □. Determine the voltage forward wave.

$$\square = 2\angle 0\ \square * (50\Omega) = 100\angle 0\ \square$$

(3-14) Under steady-state ac conditions, the forward current wave in 300□ lossless line is □ = □□∠□. Determine the current forward wave.

$$I^{\square} = \frac{\square}{\square_{\square}} = \frac{15\angle 3}{50} = 300\angle 3\ mA$$

(3-15) Under steady-state ac conditions, the reverse voltage wave on a lossless 50□ line is □ = □□□∠□ V. Determine the reverse current wave.

$$I^- = -\frac{V^-}{Z_0} = -\frac{200\angle 0}{50} = -4\angle 0 \text{ A}$$

(3-16) Under steady-state ac conditions, the reverse current wave on a lossless 75-Ω line is $I^- = 0.5\angle 2$. Determine the reverse voltage wave.

$$V^- = -Z_0 * I^- = (-0.5\angle 2) * (75) = -37.5\angle 0$$

(3-17) A table of specifications for one version of RG - 8/U 50-Ω coaxial cable indicates that the attenuation per 100 ft. at 50 MHz is 1.2 dB. At this frequency, determine the following:

(a) Attenuation factor in decibel per foot

$$\frac{1.2 \text{ dB}}{100} = 0.012 \text{ dB/Foot}$$

(b) Attenuation factor in nepers per foot

$$\frac{0.012 \text{ dB}}{8.686} = 1.382 * 10^{-3} \text{ Np/ft}$$

For a length of 300 ft. determine the following:

(c) Total attenuation in decibel

$$L_{dB} = \alpha_{dB} * d = 0.012 * 300 = 3.6 \text{ dB}$$

(d) Total attenuation in nepers

$$L = \alpha * d = (1.3815 * 10^{-3}) * 300 = 0.4145 \text{ nepers}$$

(e) V_2/V_1 Ratio using both decibels and nepers for a single wave

$$\frac{V_2}{V_1} = e^{-L} = e^{-0.4145} = 0.6607$$

In terms of decibels, this ratio may be expressed as

$$\frac{V_2}{V_1} = 10^{\frac{-L_{dB}}{20}} = 10^{\frac{-3.6}{20}} = 10^{-0.18} = 0.6607$$

(3-18) A transmission line has an attenuation of 0.05 dB/m. Determine the following:

(a) Attenuation factor in nepers/m

$$\alpha = \frac{\alpha_{dB}}{8.686} = \frac{0.05}{8.686} = 5.756 * 10^{-3} \text{ Np/m}$$

For a length of 400 m, determine the following:

(b) Total attenuation in decibel

$$L_{dB} = \alpha_{dB} * d = 0.05 * 400 = 20 \text{ dB}$$

(c) Total attenuation

$$L = Ŭ * d = (5.756 * 10^{-3})(400) = 2.303\ Np$$

(d) / Ratio using both decibels and nepers for a single wave

$$\frac{V_2}{V_1} = \quad = \quad ^{-2.303} = 0.1$$

In term of decibles:

$$\frac{V_2}{V_1} = 10^{\frac{\quad}{20}} = 10^{\frac{-20}{20}} = 10^{-1} = 0.1$$

(3-19) A single-Frequency wave is propagating in one direction on a transmission line of length 200m. with an input rms voltage of 50 V, the output rms voltage is measured as 20 V. Determine the following:

(a) Total attenuation in decibel

$$L_{dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right) = 20\log_{10}\left(\frac{50}{20}\right) = 7.959\ dB$$

(b) Total attenuation in nepers

$$L = \frac{L_{dB}}{8.686} = \frac{7.959}{8.686} = 0.9163\ Np$$

(c) Attenuation factor in decibel/meter

$$Ŭ \quad = \frac{L_{dB}}{d} = \frac{7.959}{200} = 0.03980\ dB/m$$

(d) Attenuation factor

$$Ŭ = \frac{}{} = \frac{0.9163}{200} = 4.581 * 10^{-3} \quad /$$

(3-20) A single-Frequency wave is propagating in one direction on a transmission line of length 400m. The input power to the line is 40 W, and the output power is 12 W. Determine the following:

(a) Total attenuation in decibel

$$L_{dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right) = 20\log_{10}\left(\frac{40}{12}\right) = 5.229\ dB$$

(b) Total attenuation

$$L = \frac{L_{dB}}{8.686} = \frac{5.229}{8.686} = 0.6020\ Np$$

(c) Attenuation factor in decibel/meter

$$\alpha = \frac{L_{dB}}{d} = \frac{5.229}{400} = 0.01307 \text{ dB/m}$$

(3-21) A transmission line has the following parameters at 50 MHz: = . / , = / , = / , and = / . Determine the following:

(a) Z

$$= \quad + \quad \omega L = 0.15 + \quad (2\pi * 50 \text{ MHz} * 1.2\,\mu \quad)$$

$$0.15 + \quad 376.99 \frac{\Omega}{}$$

$$377.29 \angle 1.5310 \frac{\Omega}{}$$

(b) Y

$$Y = G + \quad \omega C = 4 * 10^{-6} + j(2\pi * 50 \text{ MHz} * 10\text{pC})$$

$$4 * 10^{-6} + j3.1416 \times 10^{-3} \quad /$$

$$3.1416 * 10^{-3} \quad 1.5695 \quad /$$

(c) ,

$$\gamma = \sqrt{ZY} = \sqrt{(377.29 \quad 1.5310)(3.1416 * 10^{-3} \quad 1.5695)}$$

$$= \sqrt{1.1853 \quad 3.1005} = 1.0887 \quad 1.5503$$

$$= 22.36 * 10^{-3} + j1.088$$

$$\alpha = 22.36 \times 10^{-3} \text{ Np/m}$$

$$\beta = j1.008 \text{ rad/ft}$$

(3-22) A lossy audio-frequency line has the following parameters at 2 kHz: = . / , = . / , = / , and . Determine the following:

(a) Z

$$= \quad + \quad \omega L = 0.20 + (2\pi * 2 \text{ KHz} \times 0.1\mu)$$

$$0.20 + \quad 1.2566 \frac{\Omega}{}$$

$$0.200\angle 6.283° \ \frac{\Omega}{ft}$$

(b) Y

$$Y = G + j\omega C = 0 + j(2\pi * 2\ KHz \times 2pF)$$

$$0 + j2.5133 \times 10^{-8}\ S/ft$$

$$251.33 \times 10^{-10} \angle 1.571\ S/ft$$

(c) α, β and γ

$$\gamma = \sqrt{ZY} = \sqrt{(0.200\angle 6.283°)(251.33 \times 10^{-10}\angle 1.5701)}$$

$$= \sqrt{50.27 \times 10^{-10}\angle 1.58} = 70.9 \times 10^{-6}\angle 0.79$$

$$= 5 \times 10^{-5} + j5.029 \times 10^{-5}$$

$$\alpha = 5 \times 10^{-5}\ Np/ft$$

$$\beta = j5.029 \times 10^{-5}\ rad/ft$$

(d) Attenuation in dB/ft

$$\alpha_{dB} = 8.69 \times 5 \times 10^{-5} = 4.34 \times 10^{-4}\ \frac{dB}{ft}$$

(e) v

$$v = \frac{\omega}{\beta} = \frac{2\pi \times 2\ kHz}{5.029 \times 10^{-5}}\ 25 \times 10^{7}\ ft/s$$

(f) Z_0

$$Z_0 = \sqrt{\frac{0.200\angle 6.28°}{251.33 \times 10^{-10}\angle 1.571}} = \sqrt{8 \times 10^{6}\angle -1.57}$$

$$= 2821\angle -0.783\ \Omega = 2001 - j[illegible]1989\ \Omega$$

(3-23) A coaxial cable has the following parameters at a frequency of 1 MHz:

series resistance = 0.3 Ω/m

series reactance = 2 Ω/m

shunt conductance = 0.5 uS/m

shunt susceptance = 0.6 mS/m

Determine the following:

$$Z = 0.3 + j2\ \Omega/m = 2.022\angle 1.419\ \Omega/m$$

$$Y = 0.5\mu + j0.6m\ S/m = 0.6m\angle 1.57\ \frac{S}{m}$$

(a) α and β

$$\gamma = \sqrt{ZY} = \sqrt{(2.02\angle 1.422)(0.6\mu \angle 1.570)}$$

$$= \sqrt{1.213\mu \angle 3} = 0.0348 \angle 1.5 = 2.6m + j0.035$$

$$\alpha = 2.6m \text{ Np/m}$$

$$\beta = 0.0348 \text{ rad/m}$$

(b) Attenuation in dB/ft

$$\alpha_{dB} = 8.69 \times 2.6m = 0.02263 \text{ dB/m}$$

(c) V

$$v = \frac{\omega}{\beta} = \frac{2\pi \text{ x } 1MHz}{0.0348} = 18.1 \text{ x } 10^7 \text{ m/s}$$

(d) Z_0

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{2.02 \angle 1.4219}{0.6\mu \angle 1.57}} = \sqrt{3.37M\angle -0.148}$$

$$58.06 \angle -0.074 \ \Omega = 57.9 - j4.3 \ \Omega$$

(3-24) For the coaxial cable of problem 3-23, repeat the analysis at 100 MHz if the series resistance increases to 1 Ω/m, but the shunt conductance remains essentially the same. (Note: You must apply basic ac circuit theory to determine the new values for the reactance and susceptance.)

1. $L = \frac{X}{\omega} = \frac{2}{2\pi \text{ x } 1MHz} = 0.32 \ \mu H/m$

$$C = \frac{B}{\omega} = \frac{0.6\mu}{2\pi \text{ x } 1MHz} = 95.5 \ pF/m$$

$$Z = R + j\omega L = 1 + j2\pi \text{x} 10^8 \times 0.312 \times 10^{-6} \ \Omega/m$$

$$1 + j200 = 200 \angle 1.57 \ \Omega/m$$

$$Y = G + jB = 0.5 \text{ x } 10^{-6} + j2\pi \text{ x} 10^8 \times 95.5 \times 10^{-12}$$

$$= 0.5 \text{ x } 10^{-6} + j0.06 = 0.06 \angle 1.571 \quad S/m$$

a. $\gamma = \sqrt{ZY} = \sqrt{(200\angle 1.57)(0.06\angle 1.571)}$

$$= \sqrt{12}\angle 3.137 = 3.46 \angle 1.57$$

$$= 8.3m + j3.46$$

$\alpha = 8.3\text{m Np/m}$
$\beta = 3.46 \text{ rad/m}$

b. $\alpha_{dB} = 8.69 \times 8.3 \times 10^{-3} = 0.072 \text{ dB/m}$

c. $v = \frac{\omega}{\beta} = \frac{2\pi \times 100\text{MHz}}{3.46} = 1.814 \times 10^{8} \text{ m/s}$

d. $Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{200\angle 1.57}{0.06\ \angle 1.571}} = \sqrt{3.33 \angle -0.0048}$

$= 57.74 \angle -0.0024 = 57.74 - j0.139$

(3-25) For the circuit of fig. P3-25, determine the following:

(a) Input current I_1

$$I = \frac{E}{Z1 + Zo} = \frac{50\angle 0}{300 + (290 - j60)} = \frac{50\angle 0}{590 - j60}$$

$$= \frac{50\angle 0}{593\angle -0.1011} = 84.31\ \angle 0.1013 \text{ mA}$$

(b) Input Voltage V_1

$$V1 = Z_o * I_1 = (290 - j60) * (0.84\angle 0.101$$

$$= (296.14\angle -0.204)(0.084\angle 0.101) = 24.97\angle -0.103 \text{ V}$$

(c) Input power P_1

$$P_1 = I_1^2 * R_o = (0.084^2) * 290 = 2.06 W$$

(d) Load current I_2

$$I_2 = 0.084\angle 0.101 * (e^{-1.5}) 1\angle -0.6 = 0.019\angle -0.5$$

(e) Load Voltage V_2

$$V_2 = (24.97\angle(e^{-1.5})(1\angle -0.6) = 5.57\angle -0.703 \text{ V}$$

(f) Load Power P_2

$$P_2 = I_2^2 * R_o = (0.019^2) * 290 = 0.103 W$$

(g) Line loss in dB

$$L_{dB} = 10 \log 10 \frac{P1}{P2} = 10 \log 10 \frac{2.026}{0.103} = 13.03 \text{ dB}$$

(3-26) For the circuit of fig. P3-26, determine the following:

(a) Input current □□

$$□ = \frac{□}{□1 + □□} = \frac{80\angle 0}{□600 + □100□ + (600 + □100)} = \frac{80\angle 0}{1200 - □200}$$

(b) Input Voltage □□

Zo = Ro + jXo = 600 + j100 = 608.28 $\angle 0.165$ Ý

$□_1$ = Zo $□_1$ = $(608.28\angle 0.1652)\ (0.0658\angle -0.1652)$ = 40 $\angle 0$ V

(c) Input power □□

$$□_1 = □_1^{2} * \text{Ro} = 0.066^{2} \text{ x } 600 = 2.59 \text{ W}$$

(d) Load current □□

$$□ = \frac{□□□}{8.69} = \frac{24}{8.69} = 2.76 \text{ Np}$$

$$□_1 = (0.066\angle -0.165)(□^{□2.763})(1\angle -3) = 4.150\ □\ 10\text{^} -3 \angle -3.165\ □$$