**MAT 1475/D604: CALCULUS I**

Prof. Boyan Kostadinov
Lecture: Mondays and Wednesdays from 10-11:40 (Namm 718)
Workshop: Mondays from 12-1 PM (Namm 505A)

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**PEER LED TEAM LEARNING**

**MAT 1475 WORKSHOP SCHEDULE**

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<th>DATE</th>
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<td>February 1, 2016</td>
<td>Review Pre Calculus and begin Module I</td>
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<td>February 8, 2016</td>
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<td>February 15, 2016</td>
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<td>May 9, 2016</td>
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<td>May 16, 2016</td>
<td>Review for the Final Examination</td>
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1. Sketch a graph of the distance travelled by a student over time from the first floor to the seventh floor of City Tech’s elevator. Assume that the elevator stopped only on the 4th and 6th floors and spent approximately 3 minutes at each stop. Carefully state the assumptions you made. Does your graph represent a function?

2. Fill in the appropriate domains and ranges for each function in the table below.

<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
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<tbody>
<tr>
<td>a) $\sqrt{1-x^2}$</td>
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<td>b) $\frac{1}{1-x^2}$</td>
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<td>c) $\sin \left( \frac{1}{x} \right)$</td>
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<tr>
<td>d) $\frac{\sqrt{x}}{1-x^2}$</td>
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3. For the function \( f(x) = -2x^2 + x - 1 \), carefully evaluate and simplify \( \frac{f(x+h) - f(x)}{h} \), for \( h \neq 0 \).
4. Plot the graphs of $f(x) = \tan\left(\sin^{-1}(x)\right)$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$ separately and then on the same axes over appropriate intervals. Make a guess on the relationship between them and see if you can prove your conjecture. Identify all asymptotes?
1. Evaluate \( f(x) = \frac{x}{\sqrt{x+4} - 2} \) at several points near to \( x = 2 \) and use the result to estimate the limit \( \lim_{x \to 0} \frac{x}{\sqrt{x+4} - 2} \).
2. Simplify \( \frac{x}{\sqrt{x+4} - 2} \) and then use the simplified expression to find the limit \( \lim_{x \to 0} \frac{x}{\sqrt{x+4} - 2} \). What is the technique called? Can you think of one other way to evaluate this limit.
3. If a fair coin is tossed \( x \) times, the probability that a tail is not tossed is \( (0.5)^x \). How likely to never toss a tail is equivalent to finding \( \lim_{x \to \infty} (0.5)^x \). Estimate this limit by using several large numbers.
4. Applications discussed in problem 3 above also arise in predicting lottery outcomes and in financial mathematics to name a few. Estimate the limit \( \lim_{x \to \infty} \left( 2^{10} \left( 1 + \frac{7}{x} \right)^x \right) \) that occurs when considering compound interest.
1. Discuss whether utility bills and income tax rates are functions with discontinuities. You should create examples to justify your claims.
2. For the functions listed below, determine their points of discontinuity. State the type of discontinuity (removable, jump, infinite, or none of these) and whether the function is left-or right continuous.

a) \( f(x) = \frac{1}{x^2} \)

b) \( f(x) = 2x^{-1/5} - 4x^5 \)
3. Evaluate the limits algebraically if it exists. If not, determine whether the one-sided limits exist (finite or infinite).

a) \[ \lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2} \]

b) \[ \lim_{h \to 0} \frac{\sqrt{5 + h} - \sqrt{5}}{h} \]
4. Is the function \( f(x) = \sqrt{9 - x^2} \) continuous at \( x = 3 \)? Carefully explain your answer.
1. The position of a metal bolt falling from a skyscraper has the position function \( s(t) = -16t^2 + 19 \) in feet and time measured in seconds. Find the instantaneous velocity of the metal bolt when the time is two seconds by evaluating the limit: \( \lim_{t \to 2} \frac{s(2) - s(t)}{2 - t} \).
2. Which derivative is approximated by

\[-\cos\left(\frac{\pi}{4} - 0.00000012\right) + \frac{\sqrt{2}}{2}\]

3. State and use the **limit definition of derivatives** to compute \( f'(a) \) and find the equation of the tangent line to \( f(x) = -x + 2x^2 \) at \( a = -1 \).
4. Find the first derivatives of the functions below using the power rule and appropriate properties of derivatives.

   a) \( f(x) = 3\sqrt{x} - x^e + e^x + e^3 \)

   b) \( f(x) = x^{-4}(5x^2 - 1)^2 \)
1. **Reading assignment**: For a practical application of the product rule of derivatives see the following link. Computing the speed of model rockets with the product rule.

https://en.wikibooks.org/wiki/Calculus/Product_and_Quotient_Rules

2. State and use the Product Rule to calculate the derivative

\[
\frac{df}{dx}
\Big|_{x=9},\ f(x) = (x^{\sqrt{2}} - \sqrt{x} + 1)(x^{-2} - 3x - 1)
\]
3. State and use the Quotient Rule to calculate the derivative \( \frac{df}{dx} \), \( f(x) = \frac{5x^2 - \sqrt{x} - 2}{4x^3 + 1} \).
4. Find the rate of change of the Volume $V$ of a cylinder with respect to its radius if the height is twice the radius.

5. Find the rate of change of the fifth root $\sqrt[5]{x}$ with respect to $x$ when $x = 1, 32$ and $243$. 
6. Find the $n$-th derivative of the function $f(x) = x^k$, for the following three cases: $k < n$, $k = n$ and $k > n$. Assume that $k$ is a positive integer. The answers for the three cases are: $0$, $n!$, and $\frac{k!}{(k-n)!}x^{k-n}$ respectively. It is best if you pick appropriate values for $n$ and $k$ to see each case.
1. Compute the derivatives of the functions below.
   a) \( f(x) = \sin^2 x + e^{-x} \sec x \)

   b) \( f(x) = \sqrt{1 - \sin(x) + x^3} \)
c) \( f(x) = e^{2-5x^3} + (5x - 1)^3 \)

2. Compute the higher derivative: \( \frac{d^5}{dx^5} \left( \frac{1}{x} \right) \)
3. Evaluate the first derivatives of the following functions:
   
   a) \( f(x) = \frac{1}{x^3} \)

   b) \( f(x) = x^2 \tan^{-1}(2x) - 7 \ln(1+x^2) \)
c) \[ f(x) = \frac{(2x-1)^{11}}{(3x-1)^3} \sqrt{2x+7} e^{5x+2}. \] Hint: Do not use the quotient rule.
1. The figure below shows a portion of the graph $2x^4 - 3xy + y^2 = 20$. Find the equation of the tangent lines at the points $(1,6)$ and $(1,-3)$.
2. Find \( \frac{dy}{dx} \) given that \( \sin(\pi(x + y)) = 0 \). Sketch the graph of this equation.
3. The radius \( R \) and the height \( H \) of a circular cone change at a rate of 3cm/s. How fast is the volume of the cone increasing when \( R = 5 \) and \( H = 15 \)?
4. The base of a right triangle increases at a rate of $5 \text{ cm/s}$, while the height remains constant at $25 \text{ cm}$. How fast is the angle between its base and its hypotenuse changing when the length of its base is $25 \text{ cm}$?
1. Estimate the following using linearization:
   a) \( \sqrt{15.999} \)

   b) \( \frac{1}{\sqrt{15.999}} \)
2. Find the minimum and maximum of the functions below on the given intervals.
   
   a) \( f(x) = xe^{-x} \) on \([0, 5]\)

   b) \( f(x) = \frac{x^2 + 1}{x^2 - 1} \) on \([-5, 5]\)
3. Find the critical points and the intervals on which \( f'(x) = 2x^3 + 9x^2 + 12x + 24 \) is increasing or decreasing. Use the first derivative test to decide whether the critical point is a local maximum or minimum or neither.
1. Find the intervals on which the function \( f(x) = (x^2 - 2)e^{-x} \) for \( x > 0 \) is increasing, decreasing, concave up and concave down. Identify its inflection point, and any extrema (local max/min) and then make an appropriate sketch. Check that the horizontal axis is an asymptote to this curve.
2. Evaluate the limits below:
   a) \( \lim_{x \to \infty} (x^2 - 2)e^{-x} \)
   
   b) \( \lim_{x \to \infty} \frac{-5x^2 + 24e^{-x}}{-6x^2 + 18e^{-x}} \)
c) \[ \lim_{x \to \infty} \frac{\sqrt{9x^2 + 5}}{x} \]
1. Find the dimensions of an open box with minimal surface if its volume is $12 \text{m}^3$ and it has a square base.
2. Find the points on the arch with graph \( y = 2-x^2 \) that are closest to the point \((0,1)\).
Name: ________________________________  Points: ______

1. Evaluate the indefinite integrals below:

   a) \[ \int \left( -e^x + \frac{2}{\sqrt{x}} + \sqrt{x} \right) dx \]

   b) \[ \int \left( \frac{4x^{3/2}}{x} - 2 \right) dx \]
c) $\int \sec(x) \tan(x) \, dx$

d) $\left( \int x^{-\frac{16}{9}} + x^{\frac{9}{16}} \right) \, dx$
2. By drawing an appropriate graph of the signed area, evaluate the integrals below:
   a) \( \int_{-5}^{5} x^5 \, dx \)

   b) \( \int_{-6}^{4} |x+1| \, dx \)
c) \[ \int_{0}^{3} \sqrt{9 - x^2} \, dx \]
1. Calculate the derivatives below by using the fundamental theorem:
   a) \( \frac{d}{dx} \left( \int_1^x \frac{1}{t^3} \, dt \right) \)
   b) \( \frac{d}{dx} \left( \int_5^{\sin(x)} \sqrt{1-t^2} \, dt \right) \)
c) \[
\frac{d}{dx} \left( \int_{x^2}^{x} \sqrt{t+1} \, dt \right)
\]