MSEIP IN COMPUTING

NEW YORK CITY COLLEGE OF TECHNOLOGY

PEER LED TEAM LEARNING

MAT 1475/D604: CALCULUS I

Prof. Boyan Kostadinov

Lecture: Mondays and Wednesdays from 10-11:40 (Namm 718)

Workshop: Mondays from 12-1 PM (Namm 505A)

PEER LED TEAM LEARNING
MAT 1475 WORKSHOP SCHEDULE

DATE	MODULE
February 1, 2016	Review Pre Calculus and begin Module I
February 8, 2016	1
February 15, 2016	No workshop
February 22, 2016	2
February 29, 2016	3
March 7, 2016	4
March 14, 2016	5
March 21, 2016	6
March 28, 2016	7
April 4, 2016	8
April 11, 2016	9
April 18, 2016	10
April 25, 2016	No workshop – Spring Break
May 2, 2016	11
May 9, 2016	12
May 16, 2016	Review for the Final Examination

Name: _____ Points: _____

1. Sketch a graph of the distance travelled by a student over time from the first floor to the seventh floor of City Tech's elevator. Assume that the elevator stopped only on the 4th and 6th floors and spent approximately 3 minutes at each stop. Carefully state the assumptions you made. Does your graph represent a function?

2. Fill in the appropriate domains and ranges for each function in the table below.

Function	Domain	Range
a) $\sqrt{1-x^2}$		
b) $\dfrac{1}{1-x^2}$		
c) $\sin\left(\dfrac{1}{x}\right)$		
d) $\dfrac{\sqrt{x}}{1-x^2}$		

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3. For the function $f(x) = -2x^2 + x - 1$, carefully evaluate and simplify $\frac{f(x+h) - f(x)}{h}$, for $h \neq 0$.

4. Plot the graphs of $f(x)=\tan\left(\sin^{-1}(x)\right)$ and $g(x)=\dfrac{x}{\sqrt{1-x^2}}$ separately and then on the same axes over appropriate intervals. Make a guess on the relationship between them and see if you can prove your conjecture. Identify all asymptotes?

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Name: _____ Points: _____

1. Evaluate $f(x)=\dfrac{x}{\sqrt{x+4}-2}$ at several points near to $x=2$ and use the result to estimate the limit $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{x+4}-2}$.

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2. Simplify $\frac{x}{\sqrt{x+4}-2}\,\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$ and then use the simplified expression to find the limit $\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}$. What is the technique called? Can you think of one other way to evaluate this limit.

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3. If a fair coin is tossed x times, the probability that a tail is not tossed is $(0.5)^x$. How likely to never toss a tail is equivalent to finding $\lim_{x\to\infty}(0.5)^x$. Estimate this limit by using several large numbers.

4. Applications discussed in problem 3 above also arise in predicting lottery outcomes and in financial mathematics to name a few. Estimate the limit $\lim_{x\to\infty}\left(2^{10}\left(1+\frac{7}{x}\right)^{x}\right)$ that occurs when considering compound interest.

Name: _____ Points: _____

1. Discuss whether utility bills and income tax rates are functions with discontinuities. You should create examples to justify your claims.

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2. For the functions listed below, determine their points of discontinuity. State the type of discontinuity (removable, jump, infinite, or none of these) and whether the function is left-or right continuous.

   a) $f(x) = \frac{1}{x^2}$

   b) $f(x) = 2x^{-1/5} - 4x^5$

3. Evaluate the limits algebraically if it exists. If not, determine whether the one- sided limits exist (finite or infinite).

a) $\lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2}$

b) $\lim_{h \to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$

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4. Is the function $f(x)=\sqrt{9-x^2}$ continuous at $x=3$? Carefully explain your answer.

Name: _____ Points: _____

1. The position of a metal bolt falling from a skyscraper has the position function $s(t) = -16t^2 + 19$ in feet and time measured in seconds. Find the instantaneous velocity of the metal bolt when the time is two seconds by evaluating the limit: $\lim_{t \to 2} \frac{s(2) - s(t)}{2 - t}$.

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2. Which derivative is approximated by $\dfrac{-\cos\left(\dfrac{\pi}{4}-0.00000012\right)+\dfrac{\sqrt{2}}{2}}{0.00000012}$?

3. State and use the **limit definition of derivatives** to compute $f^{/}(a)$ and find the equation of the tangent line to $f(x)=-x+2x^2$ at $a=-1$.

4. Find the first derivatives of the functions below using the power rule and appropriate properties of derivatives.

   a) $f(x) = 3\sqrt{x} - x^e + e^x + e^2$

   b) $f(x) = x^{-4}\left(5x^2 - 1\right)^2$

Name: _____ Points: _____

1. **Reading assignment**: For a practical application of the product rule of derivatives see the following link. Computing the speed of model rockets with the product rule.
   https://en.wikibooks.org/wiki/Calculus/Product_and_Quotient_Rules

2. State and use the Product Rule to calculate the derivative
   $\left.\frac{df}{dx}\right|_{x=9}$, $f(x)=\left(x^{\sqrt{2}}-\sqrt{x}+1\right)\left(x^{-2}-3x-1\right)$

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3. State and use the Quotient Rule to calculate the derivative $\left.\frac{df}{dx}\right|_{x=1}$, $f(x)=\frac{5x^2-\sqrt{x}-2}{4x^3+1}$

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4. Find the rate of change of the Volume V of a cylinder with respect to its radius if the height is twice the radius.

5. Find the rate of change of the fifth root $\sqrt[5]{x}$ with respect x when $x = 1, 32$ and 243.

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6. Find the n-th derivative of the function $f(x) = x^k$, for the following three cases: $k < n$, $k = n$ and $k > n$. Assume that k is a positive integer. The answers for the three cases are: $0, n!$ and $\frac{k!}{(k-n)!}x^{k-n}$ respectively. It is best if you pick appropriate values for n and k to see each case.

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Name: _____ Points: _____

1. Compute the derivatives of the functions below.
   a) $f(x) = \sin^2 x + e^{-x} \sec x$

   b) $f(x) = \sqrt[3]{1 - \sin(x) + x^3}$

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c) $f(x) = e^{2-5x^3} + (5x-1)^3$

2. Compute the higher derivative: $\dfrac{d^5}{dx^5}\left(\dfrac{1}{x}\right)$

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3. Evaluate the first derivatives of the following functions:

   a) $f(x) = x^{\frac{1}{x}}$

   b) $f(x) = x^2 \tan^{-1}(2x) - 7\ln(1+x^2)$

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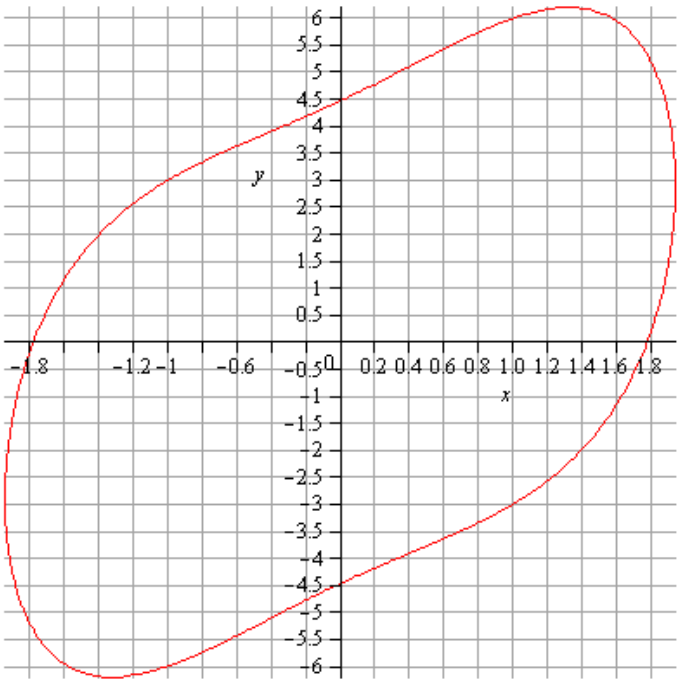
c) $f(x)=\dfrac{(2x-1)^{11}\sqrt{2x+7}}{(3x-1)^{3}}e^{5x+2}$. Hint: Do not use the quotient rule.

Name: _____ Points: _____

1. The figure below shows a portion of the graph $2x^4 - 3xy + y^2 = 20$. Find the equation of the tangent lines at the points $(1,6)$ and $(1,-3)$.



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2. Find $\frac{dy}{dx}$ given that $\sin\left(\pi(x+y)\right)=0$. Sketch the graph of this equation.

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3. The radius R and the height H of a circular cone change at a rate of 3cm/s. How fast is the volume of the cone increasing when $R = 5$ and $H = 15$?

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4. The base of a right triangle increases at a rate of $5\ cm/s$, while the height remains constant at $25cm$. How fast is the angle between its base and its hypotenuse changing when the length of its base is $25cm$?

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[MODULE 8: LINEARIZATION, EXTREMES AND MONOTONICITY ALL TOGETHER]

Name: _____ Points: _____

1. Estimate the following using linearization:
   a) $\sqrt{15.999}$

   b) $\dfrac{1}{\sqrt{15.999}}$

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2. Find the minimum and maximum of the functions below on the given intervals.
   a) $f(x) = xe^{-x}$ on $[0,5]$

   b) $f(x) = \dfrac{x^2+1}{x^2-1}$ on $[-5,5]$

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3. Find the critical points and the intervals on which $f(x) = 2x^3 + 9x^2 + 12x + 24$ is increasing or decreasing. Use the first derivative test to decide whether the critical point is a local maximum or minimum or neither.

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[MODULE 9: CONCAVITY, LIMITS, AND CURVE SKETCHING IN A TWIST]

Name: _____ Points: _____

1. Find the intervals on which the function $f(x)=\left(x^2-2\right)e^{-x}$ for $x>0$ is increasing, decreasing, concave up and concave down. Identify its inflection point, and any extrema(local max/min) and then make an appropriate sketch. check that the horizontal axis is an asymptote to this curve.

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2. Evaluate the limits below:

a) $\lim_{x\to\infty}\left(x^2-2\right)e^{-x}$

b) $\lim_{x\to\infty}\dfrac{-5x^2+24e^{-x}}{-6x^2+18e^{-x}}$

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c) $\lim_{x \to \infty} \frac{\sqrt{9x^2+5}}{x}$

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Name: _____ Points: _____

1. Find the dimensions of an open box with minimal surface if its volume is $12\,m^3$ and it has a square base.

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2. Find the points on the arch with graph $y = 2 - x^2$ that are closest to the point $(0,1)$.

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Name: _____ Points: _____

1. Evaluate the indefinite integrals below:

a) $\int\left(-e^{x}+\frac{2}{\sqrt{x}}+\sqrt{x}\right)dx$

b) $\int\left(\frac{4x^{3/2}-2}{x}\right)dx$

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c) $\int \sec(x)\tan(x)\,dx$

d) $\left(\int x^{-\frac{16}{9}} + x^{\frac{9}{16}}\right)dx$

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2. By drawing an appropriate graph of the signed area, evaluate the integrals below:

   a) $\int_{-5}^{5} x^5 \, dx$

   b) $\int_{-6}^{4} |x+1| \, dx$

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c) $\int_0^3 \sqrt{9-x^2}\,dx$

Name: _____ Points: _____

1. Calculate the derivatives below by using the fundamental theorem:

a) $\frac{d}{dx}\left(\int_1^x \frac{1}{t^4}\,dt\right)$

b) $\frac{d}{dx}\left(\int_5^{\sin(x)} \sqrt{1-t^2}\,dt\right)$

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c) $\dfrac{d}{dx}\left(\int_{x}^{x^4} \sqrt{t+1}\, dt\right)$