MAT1575 Module 9 – An algorithm for computing Riemann sums.

Objectives: Construct an algorithm for approximating definite integrals using Riemann sums.

1. Divide the interval $[a, b]$ into n equal parts. Construct a formula for Δx, which is the width of each subdivision.

2. Let $x_0 = a, x_1, x_2, \ldots x_{n-1}$ represent the left endpoint of each subdivision from question 1. Write a formula for x_i in terms of a, i, and Δx. (See Figure 1.) (Hint: The formula is an arithmetic progression.)

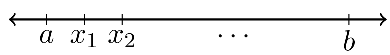
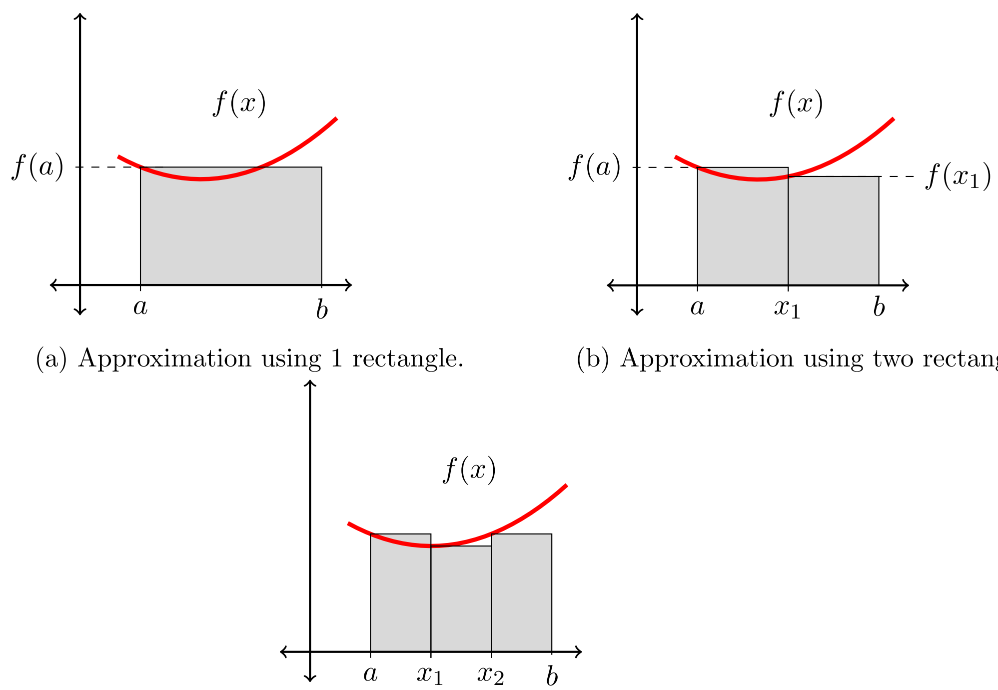


Figure 1: Dividing the interval $[a, b]$ into n equal-sized parts.



(a) Approximation using 1 rectangle.

(b) Approximation using two rectangles

(c) Approximation using three rectangles.

Figure 2: Riemann sum approximations for the area under the curve (using left endpoints).

3. Using Figure 2 and your answers to questions 1 and 2, estimate the value of $\int_a^b f(x)\,dx$ as a sum of the area of n rectangles.

4. How does your approximation in question 3 compare to the true value of $\int_a^b f(x)\,dx$ as n increases?

5. Implement your algorithm in python using trinket.io.

6. Test your algorithm against the following examples:

   (a) Approximate the area under $f(x) = x^2 - 1$ from $x = 3$ to $x = 5$.

   (b) Approximate the area under $f(x) = \cos(x)$ from $x = 0$ to $x = \frac{\pi}{2}$.

   (c) Approximate the area under $f(x) = 3e^x$ from $x = 2$ to $x = 6$.

   (d) Compute $f(x) = \int_0^x \sin(t^2)\,dt$ for different values of x.

   (e) Approximate the value of $\int_{-\infty}^{\infty} e^{-\frac{1}{2}t^2}\,dt$.