MAT1575 Module 7 – Alternating series.

Objectives: Approximate the value of π using an alternating series.

1. State the Alternating Series Test.

2. Consider the alternating series
$$\sum_{n=0}^{\infty}(-1)^n a_n = a_0 - a_1 + a_2 - a_3 + \cdots$$
where the a_n's are positive, DECREASING, and $\lim_{n\to\infty} a_n = 0$. We can write the alternating sum as
$$\sum_{n=0}^{\infty}(-1)^n a_n = \sum_{n=0}^{2k}(-1)^n a_n + (a_{2k+1} - a_{2k+2}) + (a_{2k+3} - a_{2k+4}) + \cdots$$
where $2k$ is an even number. Are the terms in parenthesis ever positive? Use this idea to show that the sum of the alternating series terms from $n = 0$ to $n = 2k+1$ differs (in absolute value) from the true value of the alternating series by at most a_{2k+2}. (Hint: Throw away all the terms after than a_{2k+2} using an inequality involving 0.)

3. Repeat the above argument with the finite sum from $n = 0$ to $n = 2k - 1$ (instead of $2k$) to show that the sum of the alternating series terms from $n = 0$ to $n = 2k$ differs (in absolute value) from the true value of the alternating series by at most a_{2k+1}.

4. The answers to questions 2 and 3 show that
$$\left|\sum_{n=0}^{\infty}(-1)^n a_n - \sum_{n=0}^{M}(-1)^n a_n\right| \leq \underline{\qquad}?$$

5. Use your answer to question 4 together with the known identity:
$$\pi = \sqrt{12}\sum_{n=0}^{\infty}\frac{(-1)^n}{3^n(2n+1)}$$
to approximate π (with an error estimate) to 5 decimal places using python. A basic skeleton of the algorithm appears here: `https://trinket.io/python/cb1db188eb` (Hint: If you want to add the first 10 terms together, then your M value must be 9 in question 4, be careful with your index values.)

6. What is the smallest number of terms do you need to sum to approximate π to 11 decimal places? DO NOT GUESS, use the work above to guarantee that your answer is right!