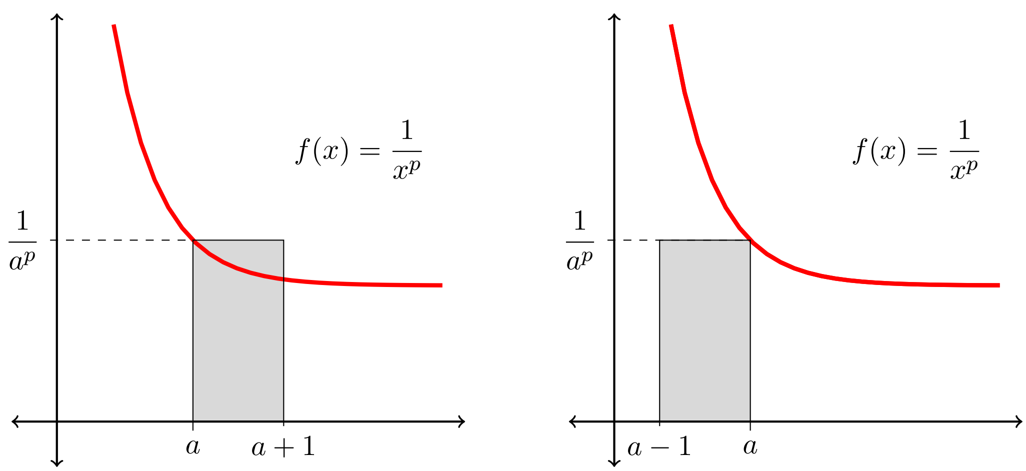
MAT1575 Module 6 – Comparing improper integrals and series.

Objectives: Construct an algorithm for estimating the value of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ when $p > 1$. Implement the algorithm in python using trinket.io.

1. Write the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$, where $p > 1$, as a sum from $n = 1$ to $n = M$ plus a sum from $n = M + 1$ to infinity.



(a) Lower bound for sum in terms of an integral.

(b) Upper bound for sum in terms of an integral.

Figure 1: Bounding a sum in terms of an integral.

2. Compute a formula for the area of the shaded rectangle in Figure 1a and a formula (as an integral) for the area under the curve from a to $a + 1$. Which value is larger?

3. Compute a formula for the area of the shaded rectangle in Figure 1b and a formula (as an integral) for the area under the curve from $a - 1$ to a. Which value is larger?

4. Use your answer to question 2 to write a lower bound for $\displaystyle\sum_{n=M+1}^{\infty} \frac{1}{n^p}$ using an improper integral. Be careful with the bounds of integration!

5. Use your answer to question 3 to write a upper bound for $\displaystyle\sum_{n=M+1}^{\infty} \frac{1}{n^p}$ using an improper integral. Be careful with the bounds of integration!

6. Use your answers to questions 1, 4, and 5 to write lower and upper bounds for $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ in terms of a finite sum from $n = 1$ to $n = M$ plus an improper integral.

7. Implement your algorithm in python using trinket.io. A basic skeleton of the algorithm appears here: `https://trinket.io/library/trinkets/6fdf7b72ba` (Hint: You might want to revisit your code from Module 4.)

8. Test your algorithm against the following examples:

   (a) Verify by approximation that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ using $M = 10, 20, 50$.

   (b) Verify by approximation that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$ using $M = 10, 20, 50$.

   (c) Verify by approximation that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} = 1.2020569031...$ using $M = 10, 20, 50$.