Solutions to 1190 Review

Problem 1

a.

Value in 2018: $\$17,000$
Value in 2019: $\$15,500$
Value in 2020: $\$14,000$
Value in 2021: $\$12,500$
Value in 2022: $\$11,000$.

b.

Just do it.

c.

Slope of this line is $-1,500$; this tells you the rate at which the car's value decreases per year.

d.

$y = 17,000 - 1,500x$

e.

18 months is 1.5 years.
Use the above formula with $x = 1.5$ to compute
$y(1.5) = 17,000 - 1,500(1.5) = 14,750$ (dollars).

f.

Solve $y(x) = 10,000$ for x.
Using the formula, this equation is
$10,000 = 17,000 - 1,500(x)$.
Rearrange to get
$x = \dfrac{17,000 - 10,000}{1,500} = 4.67$ years. This is the same as 56 months.

Problem 2

a.

Cost at mile 0: $2.50
Cost at mile 1: $4.50
Cost at mile 2: $6.50
Cost at mile 3: $8.50
Cost at mile 4: $10.50

b.

Just do it.

c.

The sequence is linear.

d.

The y-intercept is 2.5; this is the initial cost.

e.

The slope is 2; this tells you the cost increase per mile ($\$/mi$) or in other words as the rate at which the cost changes per mile.

f.

$y = 2.5 + 2x$

g.

$y(50) = 2.5 + 2(50) = 102.5$ (dollars).

h.

Solve for x in the equation
$23 = 2.5 + 2x$.
Rearrange this equation to get
$x = \dfrac{23 - 2.5}{2} = 10.25$ miles.

Problem 3

Option 1: Simple interest at rate $r = 3\% = 0.03$. The formula for the amount A in dollars after t years is
$A(t) = 1000(1 + 0.03t)$. Thus
Year 0: $A(0) = 1000(1 + 0) = 1000$ dollars
Year 1: $A(1) = 1000(1 + 0.03) = 1030$ dollars
Year 2: $A(2) = 1000(1 + 0.06) = 1060$ dollars
Year 3: $A(3) = 1000(1 + 0.09) = 1090$ dollars
Year 4: $A(4) = 1000(1 + 0.12) = 1120$ dollars.

Option 2: $r = 2\% = 0.02$ compounded monthly. The formula for the amount A in dollars after t years is
$A(t) = 1000(1 + \dfrac{0.02}{12})^{12t}$. Thus
Year 0: $A(0) = 1000(1 + \frac{0.02}{12})^{0} = 1000$ dollars
Year 1: $A(1) = 1000(1 + \frac{0.02}{12})^{12} = 1020.18$ dollars
Year 2: $A(2) = 1000(1 + \frac{0.02}{12})^{24} = 1040.78$ dollars

Year 3: $A(3) = 1000(1 + \frac{0.02}{12})^{36} = 1061.78$ dollars
Year 4: $A(4) = 1000(1 + \frac{0.02}{12})^{48} = 1083.21$ dollars.

Option 3: $r = 2.5\% = 0.025$ compounded semi-annually (i.e. twice per year). The formula for the amount A in dollars after t years is
$A(t) = 1000(1 + \dfrac{0.025}{2})^{2t}$. Thus
Year 0: $A(0) = 1000(1 + \frac{0.025}{2})^{0} = 1000$ dollars
Year 1: $A(1) = 1000(1 + \frac{0.025}{2})^{2} = 1025.16$ dollars
Year 2: $A(2) = 1000(1 + \frac{0.025}{2})^{4} = 1050.95$ dollars
Year 3: $A(3) = 1000(1 + \frac{0.025}{2})^{6} = 1077.38$ dollars
Year 4: $A(4) = 1000(1 + \frac{0.025}{2})^{8} = 1104.49$ dollars.

Option 1 yields highest earnings.

Problem 4

a.

Spider-Man seems to need life insurance more than the other characters.

b.

Marge Simpson seems to need lift insurance less than the other characters.

c.

Batman's slice takes up 18% of the pie.
Since $100\% = 360^o$ we see that
$1\% = \frac{360}{100}^0 = 3.6^0$. So every single percentage contributes 3.6^0.
Therefore $18\% = 18 \times 1\% = 18 \times 3.6^o = 64.8^o$.
So Batman's slice forms an angle of 64.8^o.

d.

Harry Potter is 15% and Marge Simpson is 11%.
That means that 26% of people surveyed choice Harry Potter or Marge Simpson.
Using the above method, we know that $1^{\%} = 3.6^o$ and therefore we find that
$26\% = 26 \times 3.6^o = 93.6^o$.

e.

16% out of 1000 people chose Fred Flintstone.
Since 100% is all 1000 people, we know that
1% is $\frac{1000}{100} = 10$ people. Therefore
$16\% = 16 \times 1\%$ is $16 \times 10 = 160$ people (each percentage contributes 10 people).
So 160 people chose Fred Flintstone.

Problem 5

This is a simple proportion (like problem 4 e). If you are comfortable setting up proportions and solving them, you should do so. However here I'll explain the reasoning.
Since 2.1 cm corresponds to 1.70 m, we conclude that
1 cm corresponds to $\frac{1.70}{2.1} = 0.81$ m.
Therefore, $8.1 = 8.1 \times 1$ cm corresponds to $8.1 \times 0.81 = 6.56$ m (each cm contributes 0.81 meters).
So the tree is 6.6 meters.

Problem 6

a.

Since you are saving 40% of the original price, this means that you actually pay 60% of the original price.
Therefore, we know that 60% of the original price P is \$180. As an equation, this says
$\frac{60}{100} \times P = \$180$. Solving for the original price P we find
$P = 180 \times 10060 = \$300.$.

(its good to check. If the original price is \$300, then 40% of \$300 is $\frac{40}{100} \times 300 = \$120$. So you save \$120 from \$300, which results in sales cost of $\$300 - \$120 = \$180$. Good).

Alternatively, if the original cost is P then your savings is $40\% \times P = \frac{40}{100} \times P$.
Thus you end up paying $\$180 = P - \frac{40}{100}P$. Solve this equation for P and you'll find $P = \$180$.
(Note that the work ends up being the same since
$180 = P - \frac{40}{100}P$. Simplifying the RHS by giving a common denominator, we have
$180 = P - \frac{40P}{100} = \frac{100P}{100} - \frac{40P}{100} = \frac{60P}{100} = \frac{60}{100} \times P$.
This give the same equation from above: $180 = \frac{60}{100} \times P$
So indeed "saving 40%" is the same as "spending 60%")

b.

After part a. part b. is much simpler.
The coupon allows you to save 10% on the (sales) price of \$180.
But 10% of \$180 is exactly $\frac{10}{100} \times 180 = \$18$.
So you have an additional \$18, and therefore the new price is
$\$180 - \$18 = \$162$.

Alternatively, saving 10% on \$180 is the same as spending 90% on \$180.
So the amount you end up spending is $\frac{90}{100} \times 180 = \$162$.

c.

The sales tax is 6.615% of \$162 which is exactly $\frac{6.615}{100} \times 162 = \$10.72$.
So the total cost (including tax) is $\$162 + \$10.72 = \$172.72$.

Problem 7.

a.

In general, men spend more than women.

b.

The biggest discrepancy in spending between men and women is spending on electronics and computers.

c.

The lowest discrepancy in spending between men and women is spending on dorm or apartment furnishings.

d.

Students spent the most on electronics and computers.
For a male student, spending in this category is \$533.17.
For a femael student, spending in this category is \$344.90.

e.

The total amount that a male student spends is $207.46 + 107.22 + 86.85 + 533.17 + 266.69 =$ \$1201.40
The total amount that a female student spends is $198.15 + 88.65 + 81.56 + 344.90 + 266.98 =$ \$980.24.

f.

(Assuming number of female and number of male students is approximately the same);
Clothing average: $\frac{207.46 + 198.15}{2} = \$202.81$.
Shoes average: $\frac{107.22 + 88.65}{2} = \$97.94$.
School supplies average: $\frac{86.85 + 81.56}{2} = \$84.21$.
Electronics and computers average: $\frac{533.17 + 344.90}{2} = \$439.04$.
Dorm or apartment furnshings average: $\frac{266.69 + 266.98}{2} = \$266.84$.

Problem 8

a.

Living room is $10ft \times (7 + 5)ft = 120ft^2$.
Hallway is $12ft \times 5ft = 60ft^2$.
Will need $120ft^2 + 60ft^2 = 180ft^2$ to cover the living room and hallway.

b.

Since $3ft = 1yd$, we have that $(3ft)(3ft) = (1yd)(1yd)$, and therefore $9ft^2 = 1yd^2$.
Thus $180ft^2 = 20 \times 9ft^2 = 20 \times 1yd^2 = 20yd^2$.

c.

Order $200ft^2$. Need $180ft^2$.
The order contains an additional $200ft^2 - 180ft^2 = 20ft^2$.

d.

Ordering $200ft^2$, but every $1ft^2$ costs \$2.5.
The total cost is therefore $200 \times 2.5 = \$500$.

e.

Since we paid for $200ft^2$ but only used $180ft^2$, we are paying for the additional $20ft^2$ that won't be used.
So the dollar value wasted is $20 \times 2.5 = \$50$.

f.

It takes 1.2 hours to install $50ft^2$.
This means is takes $\frac{1.2}{50} = 0.024$ hours to install $1ft^2$.
Therefore it takes $180 \times 0.024 = 4.32$ hours to install all $180ft^2$.

Problem 9

Driver A: 35 miles per 30 minutes is the same at 70 miles for 60 minutes, or 70 miles per hour.
Driver B : 65 miles per hour.
Driver A is faster.

Problem 10

a.

Simple interest interest is $r = 11\% = 0.11$.
Simple interest I is $I = P \times r \times t$.
Here $P = \$500$, $r = 0.11$ and $t = 2$.
Interest is $I = \$500 \times (0.11) \times 2 = \$110$.

b.

The future value is principal plus interest: $\$500 + \$110 = \$610$.

c.

2 years is 28 months, so you would pay $\$\frac{610}{24} = \$25.41$ every month.

Problem 11

a.

Just do it. I'll write out the sample space, but I won't write out a tree diagram.

Sample space: 12 possible combinations
(chicken, cake) (chicken, ice cream) (chicken, pudding)
(fish, cake) (fish, ice cream) (fish, pudding)
(beef, cake) (beef, ice cream) (beef, pudding)
(tofu, cake) (tofu, ice cream) (tofu, pudding)

b.

Probability of choosing chicken and cake is $\frac{1}{12}$.

c.

Probability of choosing a fish dish is $\frac{3}{12} = \frac{1}{4}$.

d.

Probability of getting cake or pudding for dessert is $\frac{8}{12} = \frac{2}{3}$.

e.

Probability of **not having beef** while *having pudding* is $\frac{3}{12} = \frac{1}{4}$.

Problem 12

Both types of candles are normally distributed.
We can use basic reasoning. A jar candle will burn on average for 25 hrs, but Andrei's jar candle lasted 30 hours. So the jar candle lasted longer than the average.
On the other hand, a pillar candle will burn on average for 95 hr, but Andrei's pillar candle lasted 88 hours, which is below average.
The jar candle lasted longer than average, the pillar candle lasted shorter than average, so we conclude that the pillar candle burned faster relative to other pillar candles.