$\frac{18}{20}$

Prof. Poirier
MEDU 2010

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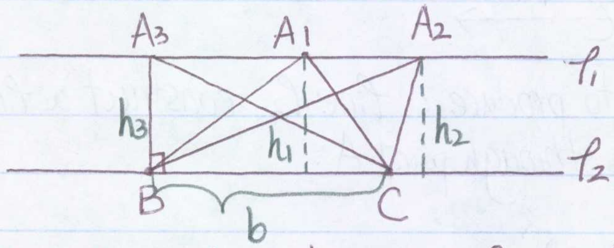
HW#4

1.

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$\ell_1 // \ell_2$, and point A is on $\ell_1$, points B and C are on $\ell_2$

A $\triangle ABC$ — area of triangle ABC remains constant ← is this your conjecture?

Assume the distance of $\overline{BC}$ is $b$, the distance between $\ell_1$ and $\ell_2$ is $h$.

$\because \ell_1 // \ell_2$

$\therefore$ The segments in between are the same length, then $h_1 = h_2 = h_3 = h = \cdots = h_n$

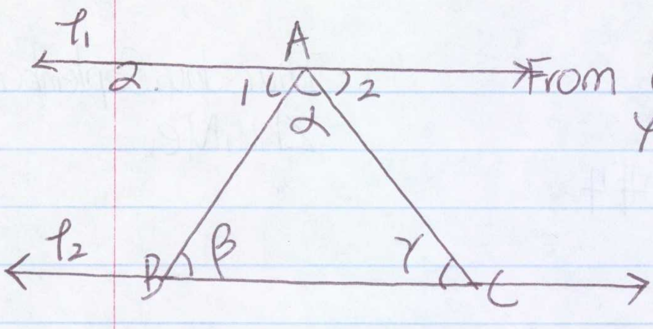
Area $\triangle A_1BC = \frac{1}{2} BC \cdot h_1$
$= \frac{1}{2} bh$

Area $\triangle A_2BC = \frac{1}{2} BC \cdot h_2$
$= \frac{1}{2} bh$

Area $\triangle A_3BC = \frac{1}{2} BC \cdot h_3$
$= \frac{1}{2} bh$

$\therefore$ Area $\triangle A_nBC = \frac{1}{2} BC \cdot h$
$= \frac{1}{2} bh$

$\therefore$ Given two parallel lines and two fixed point on one of the lines, take one point on the other line and construct a triangle, the area stays the same when we drag the single point (ex. point A shown above).




From Geogebra,
$\alpha + \beta + \gamma = 180°$ ← conjecture

Proof. Extend $\overline{BC}$ to produce a line $l_2$, construct a line $l_1$ such that $l_1 // l_2$ and $l_1$ passes through point A.

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$\because l_1 // l_2$

$\therefore m(\measuredangle 1) = m(\measuredangle \beta),\ m(\measuredangle 2) = m(\measuredangle \gamma)$ (as alternate angles)

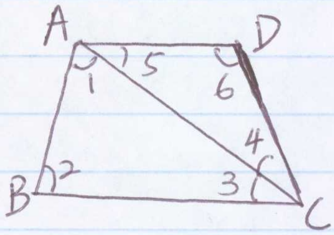
$\because \measuredangle 1, \measuredangle \alpha$, and $\measuredangle 2$ are linear

$\therefore m(\measuredangle 1) + m(\measuredangle \alpha) + m(\measuredangle 2) = 180°$

$\therefore m(\measuredangle \beta) + m(\measuredangle \alpha) + m(\measuredangle \gamma) = 180°$

$\therefore$ The sum of interior angles of a triangle is 180°. ✓

3.




From (2), we proved that the sum of interior angles of a triangle is 180°.

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Connect points A and B to construct two triangles, then

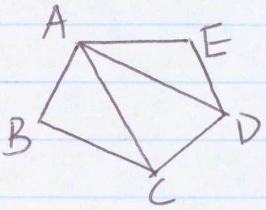
In $\triangle ABC$, $m(\measuredangle 1) + m(\measuredangle 2) + m(\measuredangle 3) = 180°$

In $\triangle ACD$, $m(\measuredangle 4) + m(\measuredangle 5) + m(\measuredangle 6) = 180°$

$180° + 180° = 360°$

$\therefore$ The sum of interior angles of a quadrilateral is 360°. ← conjecture

4.

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In a pentagon, we have 5 sides and we can construct 3 triangles.

$\therefore$ The sum of interior angles of a pentagon is $180° \times 3 = 540°$.

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5. For an n-gon, the sum of interior angles is $180°(n-2)$ ✓

proof?
(hint: generalize your previous proofs)