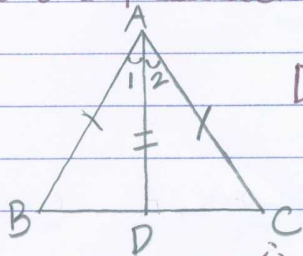Zhu, Mei

MEDU 2010

HW #2: 0.3: 1,2,3, 0.6: 1,2,3, 0.8: 1,2

0.3.1 Isosceles Triangle Theorem

Draw the angle bisector AD.
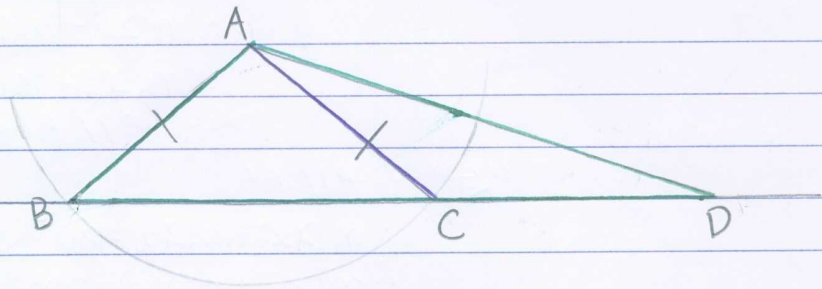
Then, $m(\angle 1) = m(\angle 2)$

$\overline{AB} \cong \overline{AC}$ (given), $\overline{AD} \cong \overline{AD}$ (reflexive property)

In $\triangle ABD$ and $\triangle ACD$, $\triangle ABD \cong \triangle ACD$ (SAS)

$\therefore m(\angle B) = m(\angle C)$, or $\angle ABC \cong \angle ACB$

$\frac{4}{4}$

0.3.2

$\frac{4}{4}$

$\triangle ABC$ is an isosceles triangle that $\overline{AB} \cong \overline{AC}$.
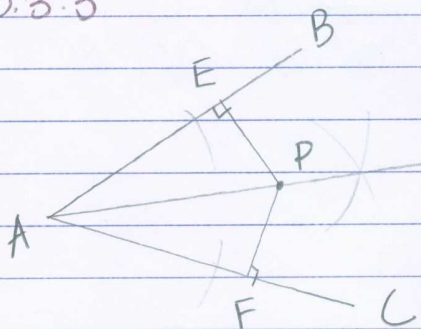
$\overline{AD} \cong \overline{AD}$ (reflexive property), $\angle D \cong \angle D$ (reflexive property)

For $\triangle ABD$ and $\triangle ACD$, we have SSA condition.

But, they are not congruent.

0.3.3

$\frac{4}{4}$

"$\exists$" $\because$ Point p is in the interior of $\angle BAC$

$\therefore m(\angle BAP) + m(\angle CAP) = m(\angle BAC)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAP) = m(\angle CAP)$

$\because PE \perp AB, PF \perp AC$

$\therefore m(\angle PEA) = m(\angle PFA) = 90°$

$\overline{AP} \cong \overline{AP}$ (reflexive property)

$\therefore \triangle PAE \cong \triangle PAF$ (AAS)

$\therefore \overline{PE} \cong \overline{PF}$

"⟸" ∵ P is in the interior of ∡BAC and the distance from P to $\overleftrightarrow{AB}$ equals the distance from P to $\overleftrightarrow{AC}$

∴ $\overline{PE} \cong \overline{PF}$, $PE \perp AB$, $PF \perp AC$

Then, △APE and △APF are right triangles.
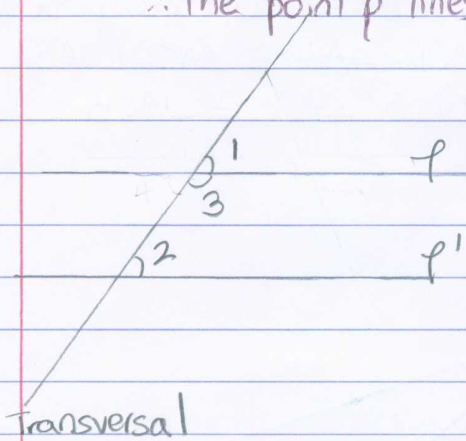
$\overline{AP} \cong \overline{AP}$ (reflexive property)

∴ △APE ≅ △APF (HL)

∴ $m(\angle EAP) \cong m(\angle FAP)$

∴ The point P lines on the bisector of ∠BAC.

0.6.1

$\frac{3}{4}$

Transversal

"⟹" ∵ Two corresponding angles are congruent and cut by the transversal

∴ $m(\angle 1) = m(\angle 2)$

∵ ∡1 and ∡3 are supplementary

∴ $m(\angle 1) + m(\angle 3) = 180°$

∴ $m(\angle 2) + m(\angle 3) = 180°$

∴ $\ell // \ell'$ ← what have you used to conclude this?

"⟸" ∵ $\ell // \ell'$

∴ $m(\angle 2) + m(\angle 3) = 180°$ ← and this?

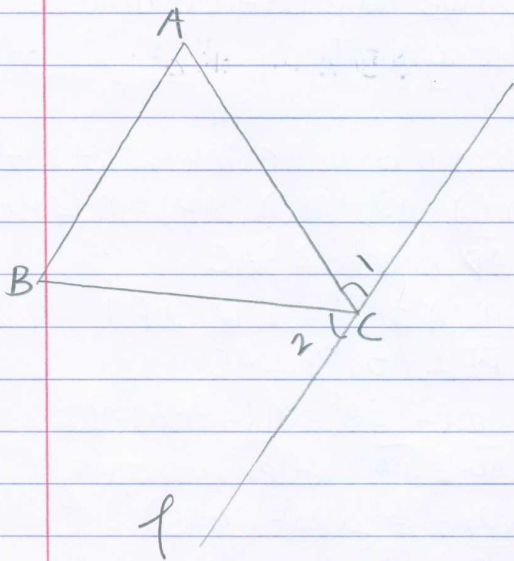∵ $m(\angle 1) + m(\angle 3) = 180°$ (supplementary angles)

∴ $m(\angle 2) = m(\angle 1)$

∴ Two corresponding angles are congruent.

0.6.2

$\frac{4}{4}$

∵ $\ell // AB$

∴ $m(\angle A) = m(\angle 1)$, $m(\angle B) = m(\angle 2)$ ✓ by alt. int. angles thm.

∵ $m(\angle 1) + m(\angle 2) + m(\angle C) = 180°$ (linear)

∴ $m(\angle A) + m(\angle B) + m(\angle C) = 180°$

good, except your notation convention might be misleading — at first I thought ∠C meant something other than the interior angle of the triangle at the vertex C.

0.6.3

"⟹"

∵ P lies on the perpendicular bisector $\overline{AB}$

∴ $\overline{AC} \cong \overline{BC}$, $PC \perp AB$ ✓

∴ $m(\angle PCA) = m(\angle PCB) = 90°$ ✓

$\overline{PC} \cong \overline{PC}$ (reflexive property) ✓

∴ $\triangle PCA \cong \triangle PCB$ (SAS) ✓

∴ $\overline{PA} \cong \overline{PB}$ ✓

$\frac{4}{4}$

"⟸" ∵ $PA = PB$

∴ $m(\angle A) = m(\angle B)$ (by exercise 0.3.1)

Assume $\overline{PC}$ is the altitude of $\triangle PAB$, then ⟵ do you just mean -drop the perpendicular from P to $\overleftrightarrow{AB}$?
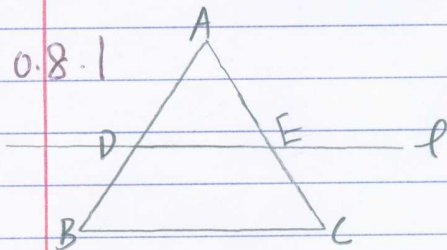
$m(\angle PCA) = m(\angle PCB)$

∵ $\overline{PC} \cong \overline{PC}$

∴ $\triangle PCA \cong \triangle PCB$ (HL)

∴ $\overline{AC} \cong \overline{BC}$

∴ PC bisects $\overline{AB}$, then $\overline{PC}$ is the perpendicular bisector

by what theorem?

0.8.1

"⟹" ∵ $\overleftrightarrow{DE} // \overleftrightarrow{BC}$

∴ $m(\angle ADE) = m(\angle ABC)$, $m(\angle AED) = m(\angle ACB)$

∴ $\triangle ADE \sim \triangle ABC$ (AA)   ok

∴ $\frac{AD}{AB} = \frac{AE}{AC}$

$\frac{3}{4}$

such that $L // \overleftrightarrow{BC}$.

"⟸" Assume that point D is on $\overline{AB}$ and ℓ passes through D. From Pasch's axiom, we know that line ℓ also intersects $\overline{AC}$ or $\overline{BC}$.

∵ $\ell // \overleftrightarrow{BC}$

∴ ℓ intersects $\overline{AC}$ but not $\overline{BC}$ at point E'

∵ $\ell // \overleftrightarrow{BC}$

∴ $\triangle ADE' \sim \triangle ABC$ and $\frac{AD}{AB} = \frac{AE'}{AC}$, $\overleftrightarrow{DE'} // \overleftrightarrow{BC}$

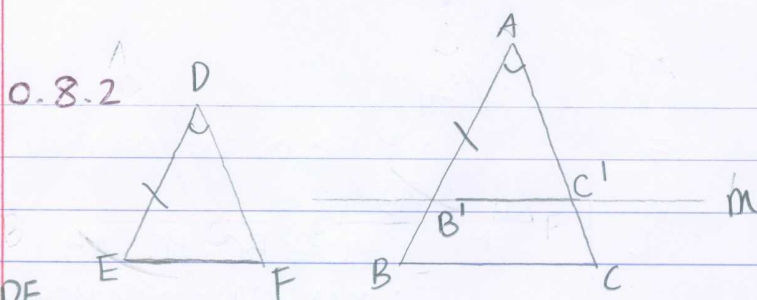∵ $\frac{AD}{AB} = \frac{AE}{AC}$ show more steps here

∴ $E' = E$, then $\overleftrightarrow{DE} // \overleftrightarrow{BC}$

so $\ell = \overleftrightarrow{DE}$ and.

0.8.2

Assume $AB > DE$

Let B' be the point between A and B such that $AB' = DE$.

Let m be the line through B' that is parellel $\overleftrightarrow{BC}$.

From Pasch's Axiom, line m also intersects $\overleftrightarrow{AC}$ or $\overleftrightarrow{BC}$

$\because$ line $m \parallel \overleftrightarrow{BC}$

$\therefore$ line m intersects $\overleftrightarrow{BC}$ at some point C' but not $\overleftrightarrow{AC}$

$\because$ line $m \parallel \overleftrightarrow{BC}$

$\therefore \triangle AB'C' \sim \triangle ABC$

$\therefore \dfrac{AB}{AC} = \dfrac{AB'}{AC'}$

$\because AB' = DE$

$\therefore \dfrac{AB}{AC} = \dfrac{DE}{AC'}$

$\because \overleftrightarrow{B'C'} \parallel \overleftrightarrow{BC}, DE = AB'$

$\therefore DF = AC'$

$\because m(\angle EDF) = m(\angle BAC)$

$\therefore \triangle DEF \cong \triangle AB'C' \ (SAS)$

$\because \triangle AB'C' \sim \triangle ABC$

$\therefore \triangle DEF \sim \triangle ABC$