MEDU 2010

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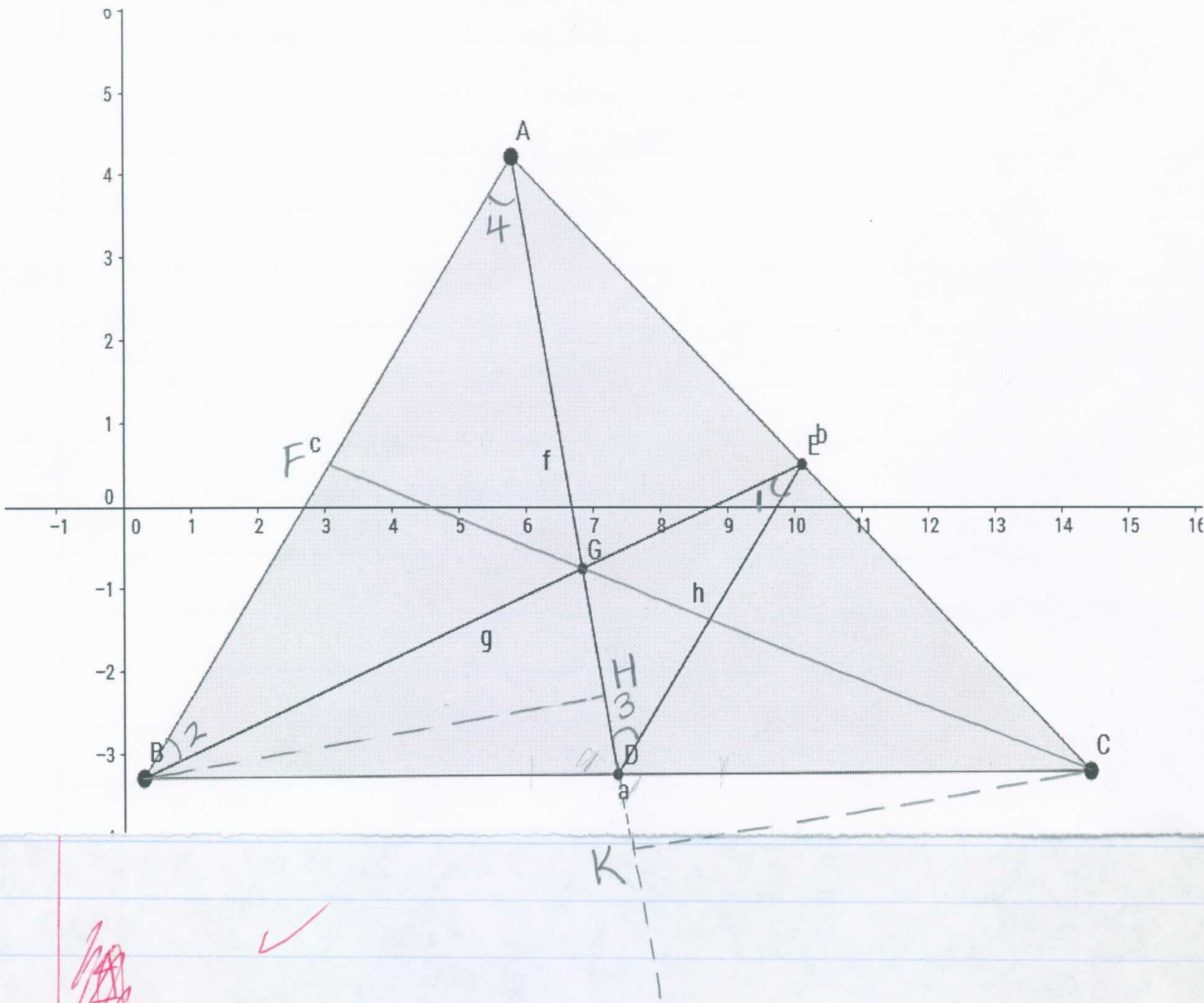
10/12

2.2.3 - 2.2.12



2.2.4

$\because$ Points E, D are the midpoints of segments $\overline{AC}$ and $\overline{BC}$ accordingly.

$\therefore \overline{ED} \parallel \overline{AB}$ ← why?

$\because \overline{ED} \parallel \overline{AB}$

$\therefore m(\measuredangle CED) = m(\measuredangle CAB), m(\measuredangle CDE) = m(\measuredangle CBA)$

Also, $m(\measuredangle C) = m(\measuredangle C)$

3/4 $\therefore \triangle CED \sim \triangle CAB$ (AAA) or (AA)

$\therefore \frac{ED}{AB} = \frac{CE}{CA} = \frac{CD}{CB} = \frac{1}{2}$ (Points E and D are midpoints)

$\therefore \frac{ED}{AB} = \frac{1}{2}$, $AB = 2ED$

2.2.5

4/4 From (2.2.4), $\overline{ED} \parallel \overline{AB}$ ok.

$\therefore m(\measuredangle 1) = m(\measuredangle 2)$, $m(\measuredangle 3) = m(\measuredangle 4)$ by which theorem

$\therefore \triangle ABG \sim \triangle DEG$ (AA)

2.2.6

$\because \triangle ABG \sim \triangle DEG$ (Proved in 2.2.5)

3/4 $\therefore \frac{GD}{AG} = \frac{GE}{BG} = \frac{ED}{AB} = \frac{1}{2}$ (Proved in 2.2.4)

almost: put 2.2.4 and 2.2.5 together

$\therefore AG = 2GD$, $BG = 2GE$, also $CG = 2GF$ as G is the centroid and $\overline{CF}$ is the median

2.2.8

Construct $\overline{BH} \perp \overline{AD}$ and $\overline{CK} \perp \overline{AD}$

$\therefore m(\measuredangle BHD) = 90^\circ$, $m(\measuredangle CKD) = 90^\circ$, $m(\measuredangle BHD) = m(\measuredangle CKD)$

$\because$ D is the midpoint

$\therefore CD = BD$

$\because$ ~~∡BH~~ $\angle BDH$ and $\angle CDK$ are vertical pair

$\therefore m(\measuredangle BDH) = m(\measuredangle CDK)$

$\therefore \triangle BHD \cong \triangle CKD$ (AAS)

$\therefore BH = CK$

$\because \alpha(\triangle ADB) = \frac{1}{2} AD \cdot BH$, $\alpha(\triangle ADC) = \frac{1}{2} AD \cdot CK$

$\therefore \alpha(\triangle ADB) = \alpha(\triangle ADC)$

2.2.9

The centroid is ~~ma~~ formed at the intersection of the medians of the triangle. We call it "the center of mass of the triangle" because the triangle can balance on that centroid point.

2.2.11

Proof.

In $\triangle ABC$, point G is the centroid, and $AE = EC$, $GE = GE$, and $GA = GC$ by the definition of centroid.

$\therefore \triangle AGE \cong \triangle CGE$ (SSS)

Similarly,

$\triangle AGF \cong \triangle BGF$, $\triangle BGD \cong \triangle CGD$

$\because \alpha(\triangle ACG) = \alpha(\triangle ACD) - \alpha(\triangle CGD)$

$\alpha(\triangle ABG) = \alpha(\triangle ABD) - \alpha(\triangle BGD)$

~~$\therefore \alpha(\triangle ACG) = \triangle A$~~

$\therefore \alpha(\triangle ACG) = \alpha(\triangle ABG)$, and $\alpha(AEG) = \alpha(EGC)$, ~~$\alpha(\triangle AEG) = \frac{1}{2}(\triangle A$~~

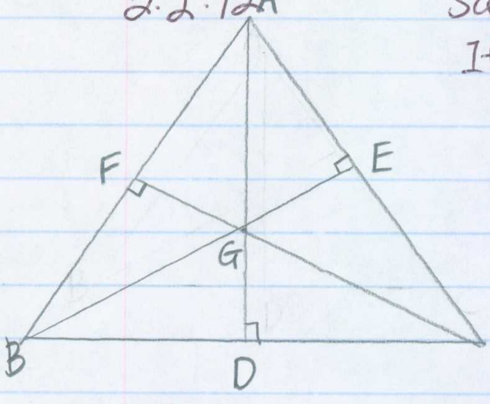
$\alpha(\triangle AEG) = \frac{1}{2}(\triangle ACG)$ $\alpha(\triangle AEG)$

$\because \alpha(\triangle AFG) = \alpha(\triangle BFG)$, $\alpha(\triangle AFG) = \frac{1}{2}\triangle(AGB) =$ ~~$(\triangle AEG$~~

$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle ECG)$

$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle ECG) = \alpha(\triangle AEG) = \alpha(\triangle CDG) = \alpha(\triangle BDG)$

2.2.12



Same procedure as in 2.2.11.

If $\triangle ABC$ is an equilateral triangle, then $\overline{AD} \perp \overline{BC}$, $\overline{BE} \perp \overline{AC}$, and $\overline{CE} \perp \overline{AB}$

$\because \triangle ABC$ is an equilateral triangle

$\therefore AB = BC = AC$

$\because$ Points D, E, and F are the median points of $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ accordingly.

$\therefore BD = DC = CE = EA = AF = FB$

Obviously,

$\alpha(\triangle AFG) = \alpha(\triangle BFG)$, $\alpha(\triangle BGD) = \alpha(\triangle CGD)$,

$\alpha(\triangle GCE) = \alpha(\triangle GAE)$

$\because$ G is the centroid of $\triangle ABC$

$\therefore GD = GE = GF$

$\therefore \alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle BGD) = \alpha(\triangle CGD) = \alpha(\triangle GCE) = \alpha(\triangle GAE)$

Converse, if $\alpha(\triangle AFG) = \alpha(\triangle BFG) = \alpha(\triangle BGD) = \alpha(\triangle CGD) = \alpha(\triangle GCE) = \alpha(\triangle GAE)$, then it is no necessary that $\triangle ABC$ is equilateral.