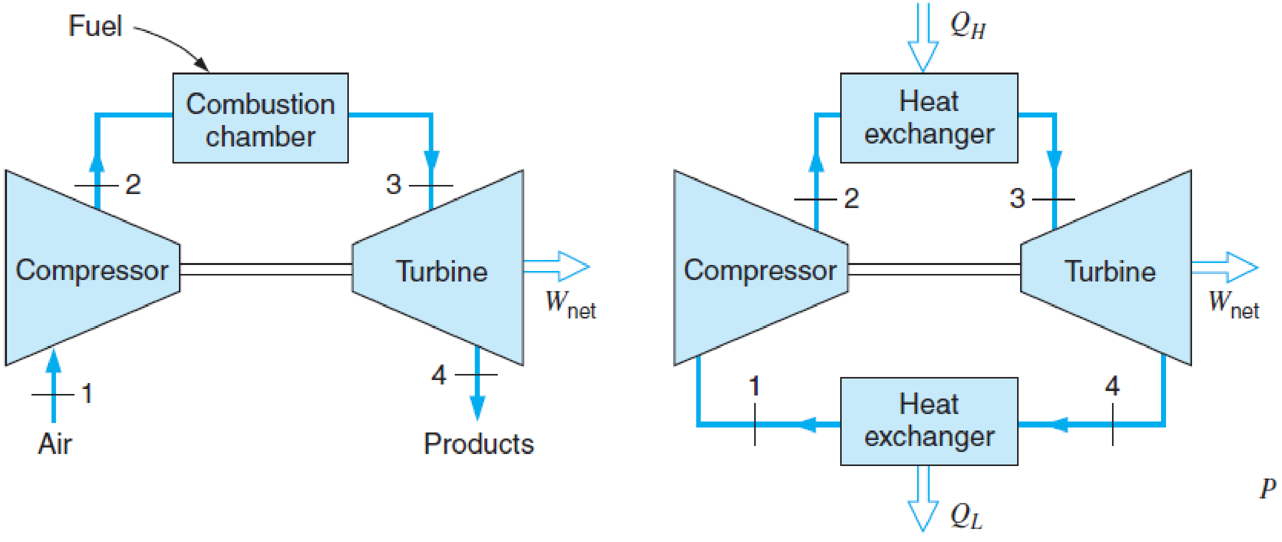
Gas Turbine and Refrigeration Cycles

BRAYTON CYCLE

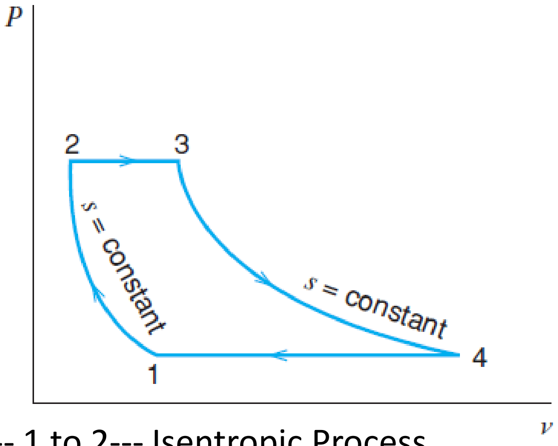
PV/T=constant<-- Ideal Gas Law
P is proportional to T, at constant V
V is proportional to T at constant P

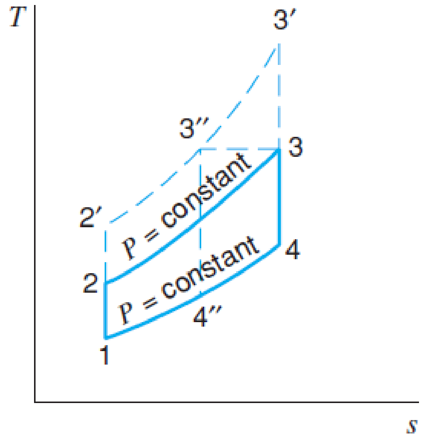
- Gas Turbine on Brayton Cycle
- Air Standard Cycle
- Open cycle– Jet Engine is an Example
- Closed Cycle
- P-V: Pressure-Volume Cycle
- T-S: Temperature-Entropy
- S= dQ/T, J/kg.K

S=Constant means the process is isentropic
P= Constant means Isobaric



(a) (b)





Consider Air as a system fluid→ Air Standard Cycle

Why is it called Gas turbine→ Because gas is a combustible media
It can be Natural Gas or Gasoline or Jet Fuel.

It's a 4-process system--- 1 to 2--- Isentropic Process
2 to 3---Isobaric process
3 to 4---Isentropic Process
4 t o1---Isobaric Process

Process Definition

- Isentropic Process- Entropy is constant, Ideally it is an insulated System, where heat loss is zero
- Isobaric Process- Process that occurs at constant Pressure

Isentropic Process

Table of isentropic relations for an ideal gas [edit]

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = \left(\frac{\rho_2}{\rho_1}\right)^{(\gamma-1)}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = \frac{V_2}{V_1} = \frac{\rho_1}{\rho_2}$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1}$$

Derived from

$$PV^{\gamma} = \text{constant},$$
$$PV = mR_sT,$$
$$P = \rho R_sT,$$

where:

- P = pressure,
- V = volume,
- γ = ratio of specific heats = C_p/C_v,
- T = temperature,
- m = mass,
- R_s = gas constant for the specific gas = R/M,
- R = universal gas constant,
- M = molecular weight of the specific gas,
- ρ = density,

Value of gamma=1.4 for ideal gas

Efficiency of Brayton Cycle

Thermal Efficiency--

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{h_4 - h_1}{h_3 - h_2} \approx 1 - \frac{C_P(T_4 - T_1)}{C_P(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Q= Heat
h=Enthalpy

From the ideal cycle we know that the pressure increase in the compressor equals the pressure decrease in the turbine, so

2 to 3→ Heat Addition, Qh=m*(h3-h2)
4 to 1→ Heat Loss, Q_L=m*(h4-h1)

$$\frac{P_3}{P_4} = \frac{P_2}{P_1}$$

and from the two isentropic processes we get the power relations as

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{k/(k-1)}$$

$$\frac{T_3}{T_4} = \frac{T_2}{T_1} \therefore \frac{T_3}{T_2} = \frac{T_4}{T_1} \quad \text{and} \quad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

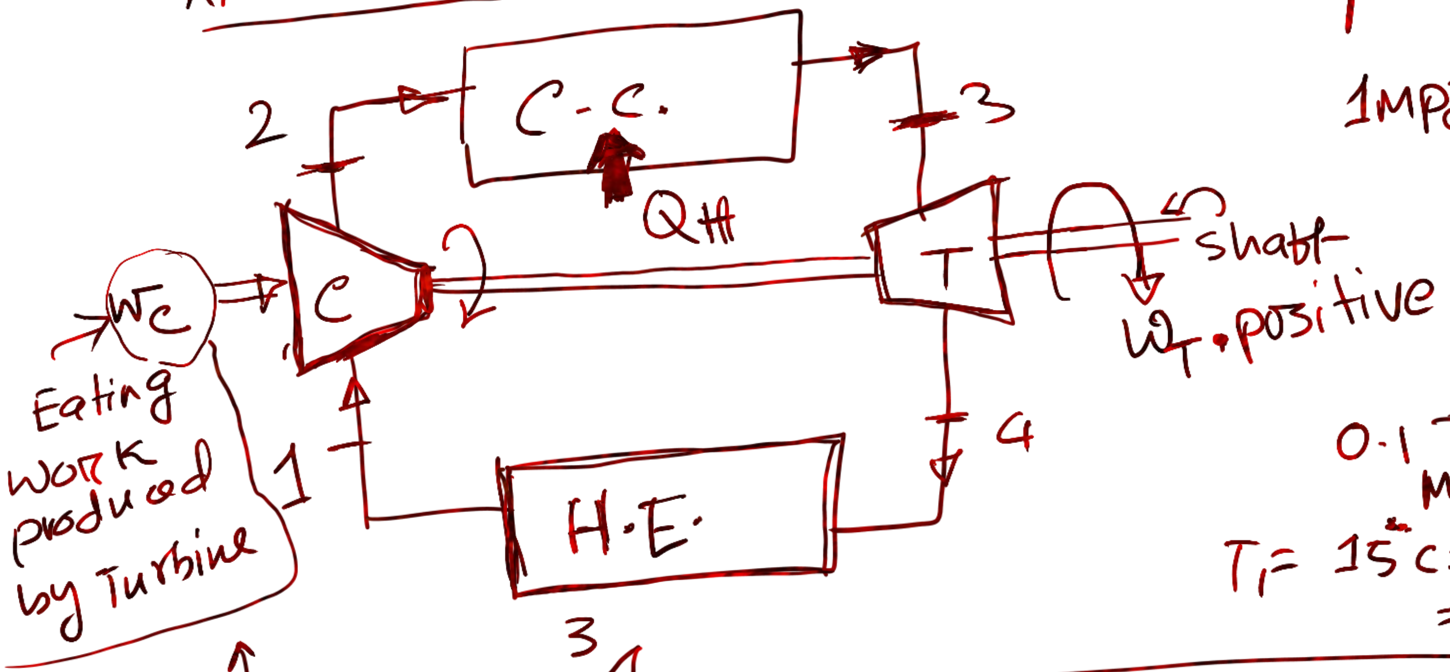
H= Cp* Delta T
Cp= Specific Heat at Constant Pressure , kJ/kg.K
Cv=Specific Heat at Constant volume , kJ/kg.K
K=Cp/Cv
Find cp of air , 1 kJ/kg.K

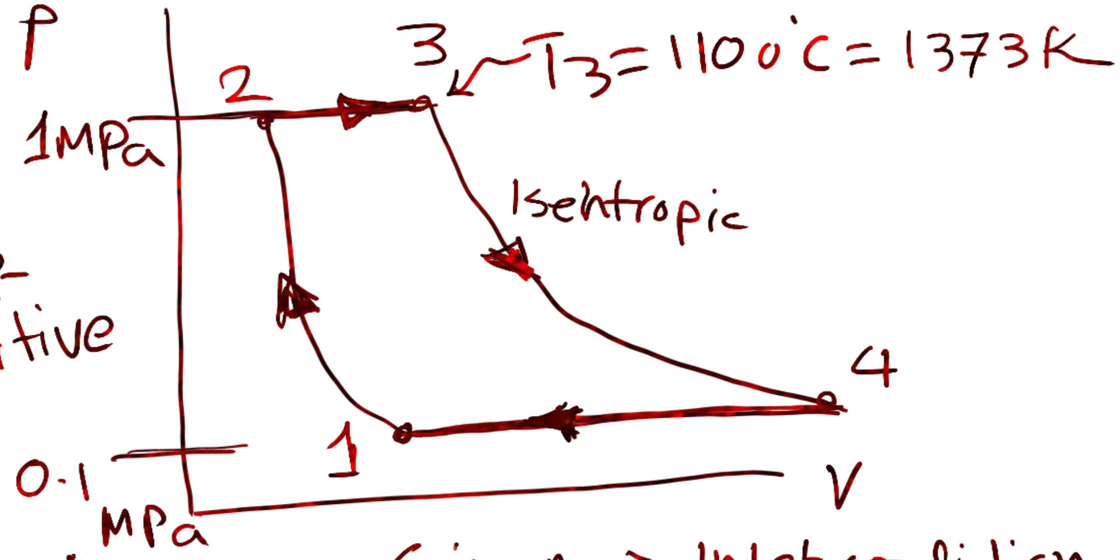
Is heat costly?
Is heat equivalent to money?
So don't you need an efficient system?

The cycle efficiency thus becomes

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}}$$

Air-standard cycle example problem:-





$T_1 = 15°C = (15+273)$
$= 288K$

Given → Inlet condition T_1, P_1

Exit → P_2

Find → P, T at each point

compressor work

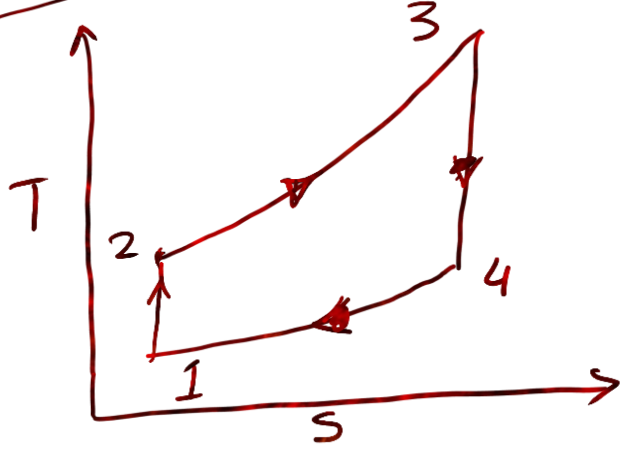
Turbine work

$\eta_{th} = ?$

$K = 1.4$

Known:- $h = c_p * T$ for gas

$W \equiv \Delta h$, kJ/kg



Is W_c positive or negative?

Work done on the system → negative

Solution:- 1→2 Isentropic process

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} = 288 * \left(\frac{1}{0.1}\right)^{\frac{0.4}{1.4}}$$

$$= 556.1\ °K$$

W_c = compressor work $= h_2 - h_1 = c_p(T_2 - T_1)$

$$W_c = h_2 - h_1 = c_p(T_2 - T_1) = 1.00 \frac{kJ}{kg \cdot K}(556 - 288)°K$$

$$= 268 \ kJ/kg$$

Let's go to Turbine side

Isentropic process $\rightarrow \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{k-1/k} \rightarrow$

$$T_4 = T_3 * \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1373 * \left(\frac{0.1}{1}\right)^{\frac{0.4}{1.4}}$$

$$= 711 \ K$$

Turbine work, $W_T = h_3 - h_4 = c_p(T_3 - T_4)$

$$= 1.00 * (1373 - 711)$$

$$= 665 \ kJ/kg$$

W_{net} = Net work by the Engine

$$= W_T - W_c = (665 - 268)$$

$$= 397 \ kJ/kg$$

Efficiency:-

Q_H = Heat Added

$$= c_p(T_3 - T_2)$$

$$= 1.00(1373 - 556)$$

$$= 820 \ kJ/kg$$

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{397}{820} = 48\% < 50\%$$

Using, $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{288}{556} = 48\%$

Use This Eqn or $\eta_{th} = \frac{W_{net}}{Q_H} = \frac{W_T - W_C}{Q_H} = \frac{W_{net}}{\text{Heat Input}}$

What is the ultimate purpose of Thermodynamic cycle?

↳ To generate work from heat.

compressor Ratio = How much air is compressed

$= \frac{P_2}{P_1} = \frac{1}{0.1} = 10$

Do It yourself ← HW # 4

A Brayton cycle → Air enters into compressor at 100 kpa and 20°c

Compressor ratio is 12:1

Max temp is 1100°c, air flow rate $\dot{m} = 10$ kg/s

C_p of air = 1.004 kJ/kg°K

Determine, compressor, Turbine work and power & thermal Efficiency of the cycle

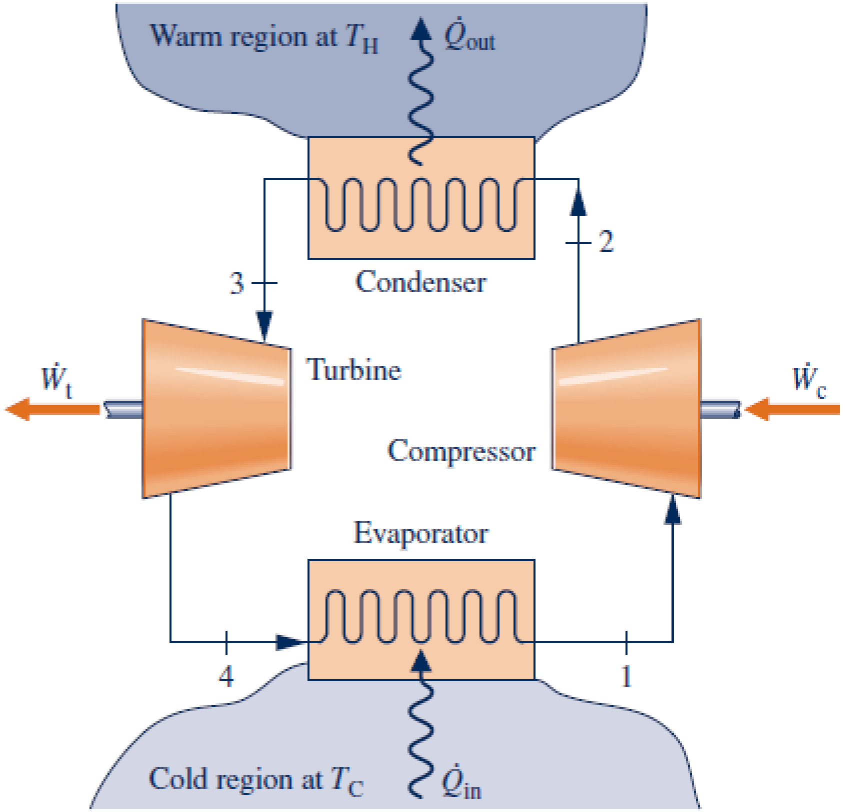
$W_c = ?$ $W_T = ?$ $\eta_{th} = ?$

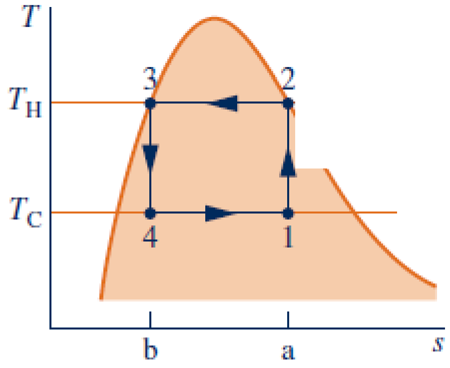
$P_c = ?$ $P_T = ?$ $Q_H = ?$

Power = kW

Power = $\dot{m} * h$ = kJ/s = $\frac{kJ}{kg} * \frac{kg}{s}$

Refrigeration Cycle-Carnot Cycle





Carnot Vapor Refrigeration Cycle

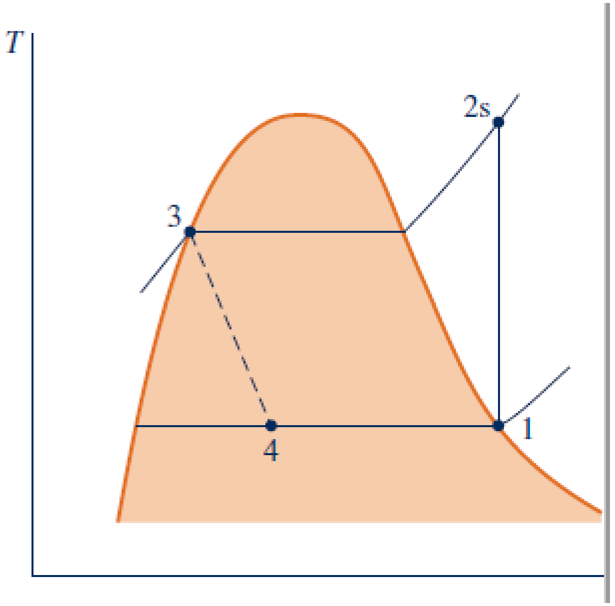
Co-efficient of Performance
= Ratio of Refrigeration effect to Net Work

$$\beta_{max} = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{\text{area } 1\text{–a–b–4–1}}{\text{area } 1\text{–2–3–4–1}} = \frac{T_C(s_a - s_b)}{(T_H - T_C)(s_a - s_b)}$$

which reduces to

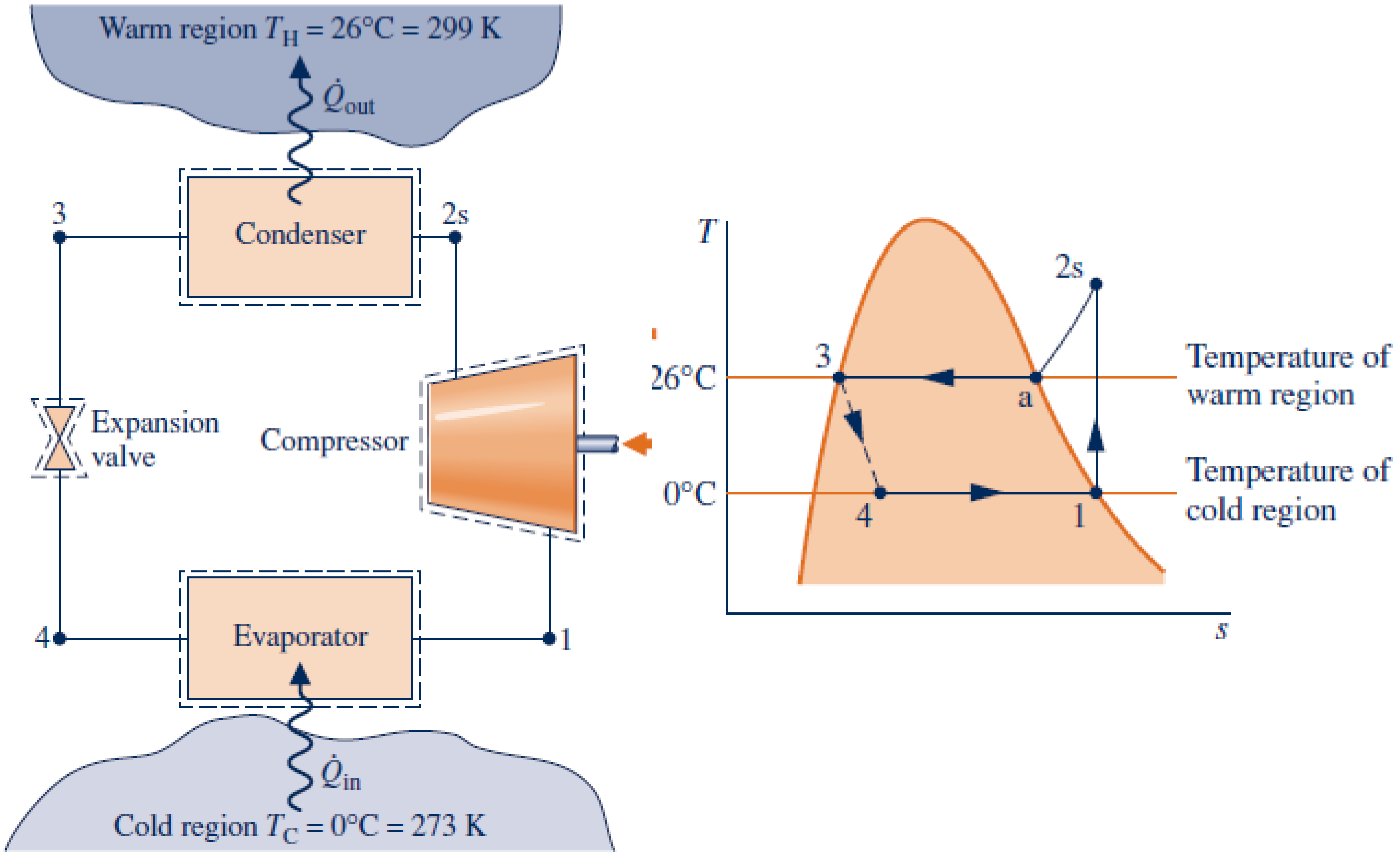
$$\beta_{max} = \frac{T_C}{T_H - T_C}$$

Ideal Vapor Compression Cycle



Ideal Vapor Compression Ref. Cycle

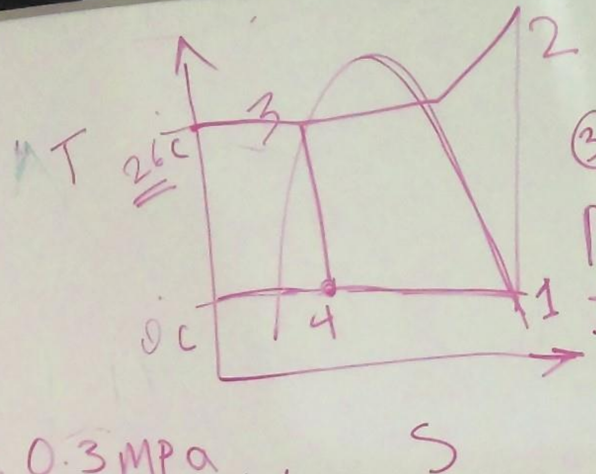
- Exa



Given-
Ref 134a

Mass flow rate 0.08 kg/s

Determine
(a) Compressor Power
(b) Refrigeration Capacity
(c) COP
(d) COP of Carnot Cycle



③ → sat. Liquid

$P_3 = 0.7$ MPa

$h_3 = 85$ kJ/kg

$s_3 = 0.33$ kJ/kgK $= s_4$

$P_1 = 0.3$ MPa

$h_1 = h_g = 247$ kJ/kg, $s_1 = 0.9$ kJ/kgK $= s_2$

h_2 (at 0.7 MPa S.H.) → 261 kJ/kg

$W_c = (h_2 - h_1) = 261 - 247 = 14$ kJ/kg

$$COP = \frac{\text{Cooling effect}}{W_c} = \frac{157.6}{14} = 11.2$$

$$= \frac{Q_{cold}}{Q_{hot} - Q_{cold}}$$

Cooling Effect

$= h_1 - h_4 = 247 - 89.4$

$= 157.6$ kJ/kg

$s_4 = 0.33$

$= 0.197 + x * (s_{fg})$

$= 0.197 + x(s_g - s_f)$

$= 0.197 + x(0.9 - 0.197)$

$$x = \frac{0.33 - 0.197}{0.9 - 0.197}$$

$= 0.2 \simeq 20\%$

$h_4 = h_f + x \times h_{fg}$

$= 50 + 0.2 * 197$

$=$ 89.4 kJ/kg

Compressor power

$= W_c * \dot{m} = 14$ kJ/kg $* 0.08$ kg/s

$= 1.1$ kW