Beam Design

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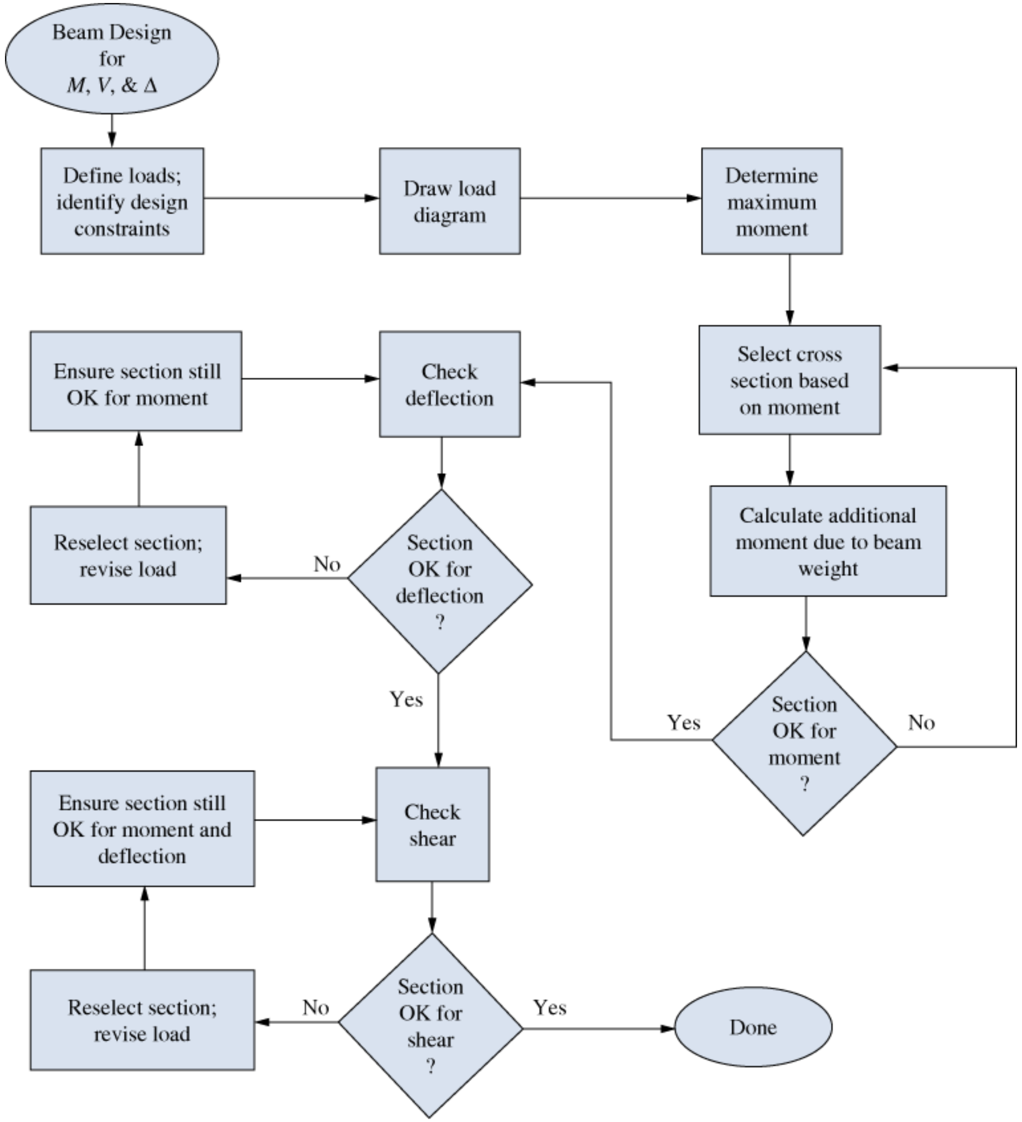
MECH 2333



- What Type of beam you need?
  - To fulfill Load condition
  - To stay within recommended deflection
- Steel Section can be W, S, I, C Beam
- Dead Load, Live Load
- Its an Iterative Process

Design of Steel Beam

- Allowable Stress= Failure Stress/Safety Factor
- Design strength=failure Strength/safety factor
- Strength Required must be less than strength furnished.

For Example, Failure Stress is 450 Mpa, Safety Factor is 2.5, then What is the Allowable stress on the beam?
Allowable stress is 180 Mpa.

divided by a safety factor. The safety factor is denoted by Ω_b and is equal to 1.67. The strength required is the applied bending moment calculated either using formulas or using shear and moment diagrams. We will denote this moment as M_a. As a limit, for design, we may then write

$$M_a = \frac{M_p}{\Omega_b} = \frac{F_y Z_x}{1.67}$$

where Z_x is the plastic section modulus with respect to the x centroidal (strong) axis of the W-shape cross section. The plastic section modulus is a tabulated quantity. Therefore, the moment design equation for minimum required Z_x is

$$\text{required } Z_x = \frac{1.67\, M_a}{F_y}$$

Considering deflection, the relationship is

calculated deflection $\leq$ deflection limit

or

$$\Delta \leq \Delta_{\text{limit}}$$

We may either check the deflection, as in the expression just shown, or determine a required minimum moment of inertia I. The deflection limit is commonly a fraction of the span length, such as span/400. For example, for a single-span, simply supported beam supporting a uniformly distributed load the midspan deflection Δ is (refer to Case 1 in Appendix H):

$$\Delta = \frac{5wL^4}{384EI}$$

For design, as a limit, $\Delta = \Delta_{\text{limit}}$, then solving for a minimum required I we have:

$$\text{required } I = \frac{5wL^4}{384E\Delta_{\text{limit}}}$$

Although shear rarely governs, it should be checked. The average web shear approach (see Section 14.6) is used and for almost all steel W-shape beams the strength relationship is written

$$V_a \leq \frac{0.6 F_y A_w}{\Omega_v}$$

where V_a = the applied shear force
F_y = the yield stress of the steel
A_w = the full-depth web area ($d \times t_w$)
Ω_v = the shear safety factor and is equal to 1.50

The preceding equation can be simplified to

$$V_a \leq 0.4 F_y d t_w$$

- Ma= Design Moment
- Zx= Plastic Section Modulus
- Ωb= Design Bending Safety Factor=1.67
- Va=Design Shear
- Aw=Full Depth Web Area (dxtw)

- The name of this Approach
  - ASD (Allowable Stress Design)

Deflection limit, for example , Span Length/360 or Span length/400

Example Problem

EXAMPLE 16.1 Select the most economical (lightest) W shape to support a superimposed uniformly distributed load of 4 kips/ft on a simply supported span of 25 ft, as shown in Figure 16.3. Assume the yield stress of the steel to be 50 ksi. The deflection limit for total load is span/360.

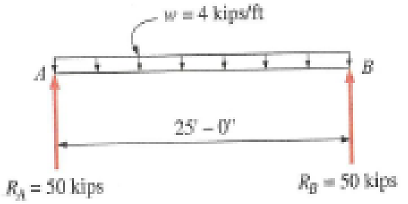


FIGURE 16.3 Load diagram and reactions.

Solution The load diagram with the reactions is shown in Figure 16.3. Since the beam is simply supported and carries a uniformly distributed load, maximum shear and moment can be computed using the shear and moment equations of Appendix H. The maximum moment is calculated from

$$M_a = \frac{wL^2}{8} = \frac{(4\ \text{k/ft})(25\ \text{ft})^2}{8} = 313\ \text{k-ft}$$

This value represents the maximum moment due to the superimposed loads and does not include the weight of the beam. The required strong-axis plastic section modulus is then obtained from

$$\text{required } Z_x = \frac{1.67\ M_a}{F_y} = \frac{1.67(313\ \text{k-ft})(12\ \text{in./ft})}{50\ \text{ksi}} = 125.5\ \text{in.}^3$$

Appendix I, Table I.1, is set up to aid in the selection of the lightest W shape that provides a given Z_x. Shapes are listed in the order of decreasing Z_x values. Some of the shapes are grouped and headed with a shape shown in boldface type. The shape at the top of each group is the lightest shape in the group. We will therefore try a W21 × 62 (Z_x = 144 in.^{3}).

- Zx equals approximately Sx+Sy

Next, add the effect of the beam weight (0.062 k/ft):

$$\text{additional } M_a = \frac{wL^2}{8} = \frac{(0.062\text{ k/ft})(25\text{ ft})^2}{8} = 4.84\text{ k-ft}$$

$$\text{total } M_a = 313\text{ k-ft} + 4.84\text{ k-ft} = 318\text{ k-ft}$$

$$\text{required } Z_x = \frac{1.67\,M_a}{F_y} = \frac{1.67(318\text{ k-ft})(12\text{ in./ft})}{50\text{ ksi}} = 127.5\text{ in.}^3$$

$$127.5\text{ in.}^3 < 144\text{ in.}^3$$

(O

Next we will calculate the required moment of inertia based on the maximum allowable deflection:

$$\Delta_{\text{limit}} = \frac{\text{span}}{360} = \frac{25\text{ ft}(12\text{ in./ft})}{360} = 0.833\text{ in.}$$

The total load is 4 k/ft + 0.062 k/ft = 4.06 k/ft:

$$\text{required } I = \frac{5wL^4}{384\,E\Delta_{\text{limit}}} = \frac{5(4.06\text{ k/ft})(25\text{ ft})^4(1728\text{ in.}^3/\text{ft}^3)}{384(30{,}000\text{ ksi})(0.833\text{ in.})} = 1428\text{ in.}^4$$

I_x for the W21 × 62 is 1330 in.^{4} Since 1330 in.^{4} < 1428 in.^{4}, the shape is N.G. for deflection.

Use Appendix J, Table J.1. This table lists W shapes in order of decreasing I_x. Entering the tal with a required I_x of 1428 in.^{4}, we select a W21 × 73 (I_x = 1600 in.^{4}). Note also that Z_x for t W21 × 73 is 172 in.^{3} and exceeds the required Z_x of 127.5 in.^{3} The increase of 11 lb/ft in the bea weight is insignificant.

Last, check shear. For the W21 × 73, d = 21.20 in. and t_w = 0.455 in. The maximum appli shear force, including the beam weight, is

$$V_a = \frac{wL}{2} = \frac{(4.07\text{ k/ft})(25\text{ ft})}{2} = 50.9\text{ k}$$

The shear capacity is

$$0.4F_y d t_w = 0.4(50\text{ ksi})(21.20\text{ in.})(0.455\text{ in.}) = 192.9\text{ k}$$

$$50.9\text{ k} < 192.9\text{ k}$$

(O.K

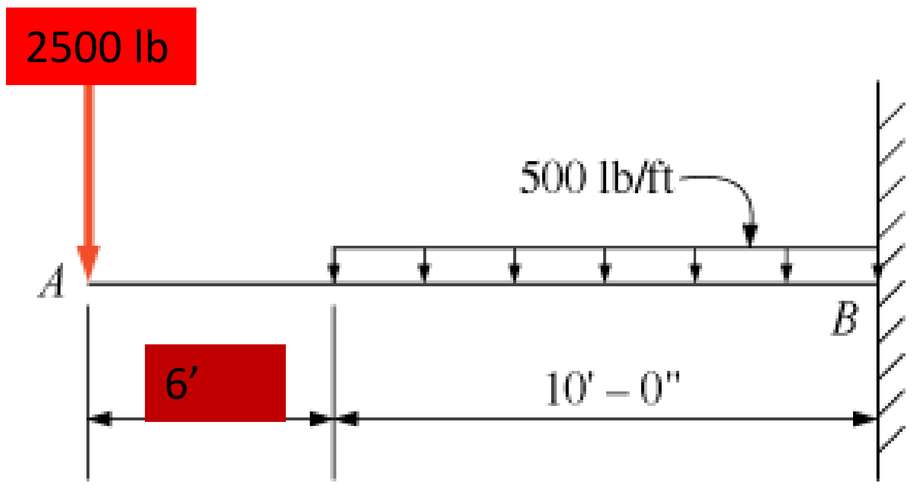
The W21 × 73 is satisfactory for moment, deflection, and shear.

Use a W21 × 73.

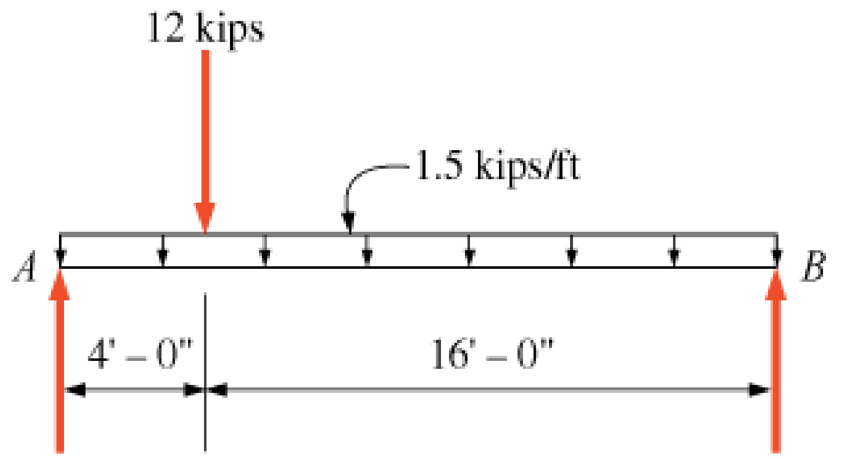
E of steel= 30 Msi
=30000 ksi
=200 GPa

- It was safe in Moment
- But was not safe in deflection
- Then Chose Deeper beam
- Then safe in deflection
- Then checked for Shear
- Then found that is safe in shear.
- Design is good.

In class Practice Problem

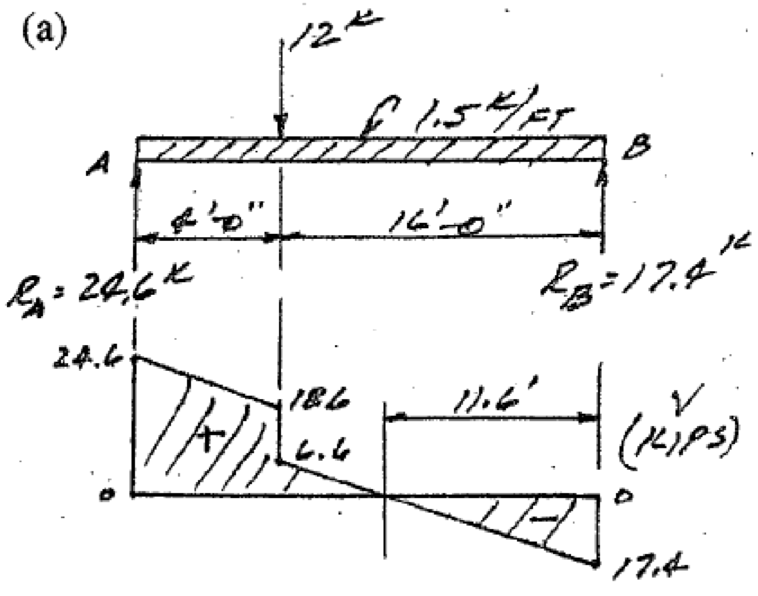


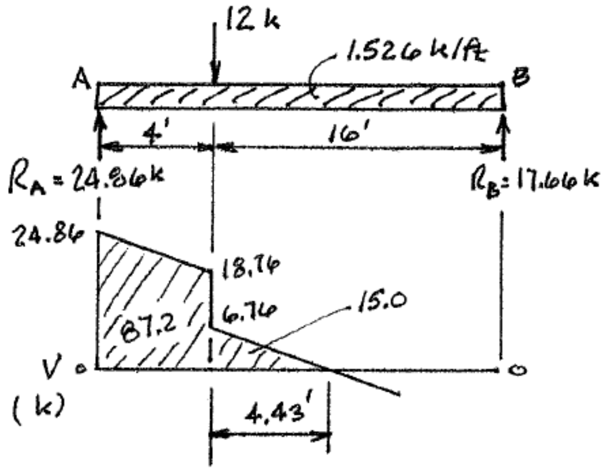
- Select the lightest W shape for the cantilever beam shown
- Consider Beam Weight.
- Assume Fy= 50 KSI
- E=30 MSI
- You need the Table for beam properties
- Equation for the Deflection-



- Design the lightest W shaped beam for the loading condition shown,
- Given Fy=50 ksi
- E=30 MSI=30000 ksi
- Deflection limit – span length/360, in/in

Prob. 16-20
(a)





$M_{max} = 100.9$ k-ft

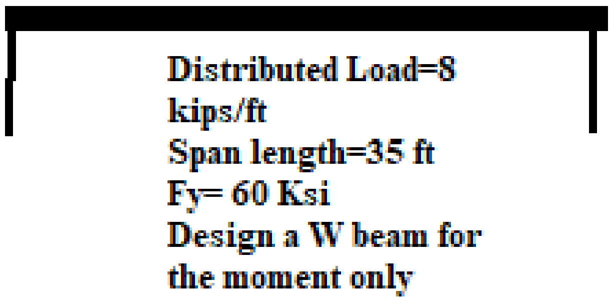
$$\text{req'd } Z_x = \frac{1.67(100.9)(12)}{50} = 40.4 \text{ in.}^3$$

Try W16x26: $Z_x = 44.2$ in.^{3}, $d = 15.7$ in., $t_w = 0.250$ in.
See new load and V diag. following:
From V diag.: new $M_{max} = 102.2$ k-ft

$$\text{req'd } Z_x = \frac{1.67(102.2)(12)}{50} = 41.0 \text{ in.}^3 \text{ (O.K.)}$$

Check shear:
$V_{max} = 24.86$ k
$V_R = 0.4(50)(15.7)(0.250) = 78.5$ k (O.K.)
Use W16x26

Class Problem



- Ma=w*L^2/8=8*35*35/8=1225 kip-ft

- Z=1.67*Ma/Fy=1.67*1225*12 (kip-in)/60=409 in^3, Look for higher Z value in the Appendix I
- Z= Section modulus
- Select beam from Appendix I , W 24x162, Zx=468 in^3, I=5170 in^4

Adding wt., 0.161 Kip/ft

Re-calculated Ma=8.161*35*35/8=1249 kip-ft

Z=1.67*1249*12/60=417 still less than 468, so **W 24x162** is successful in Moment

If we consider other criteria

Let’s try for Deflection
Deflection limit= Span length/360=35*12/360=1.16 in/in
(That means, In a 30 ft span, max deflection is 1.16 in, this is elastic deformation)

Req. I=5*w*l^4/(384*E*Deflection Limit)= 7874 in^4>5170 in^4
Req I is higher, so lets choose bigger beam, W 36x150

Calculate Shear capacity 0.4*Fy*d*tw= 0.4*60*35.9*0.625=538.5 K

Applied shear = w*L/2=143 kip < 538.5 kip, so beam is safe in shear, We can choose W 36x150, **O.K.**