# ANALYSIS OF SUSPENSION SYSTEMS

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## ***ABSTRACT:***

The objective of this report is to analyze and evaluate systems in modern automobiles by discussing the impact struts, coil overs, and compactness have on the system's vibration and frequency, as well as including how these elements affect a passenger's comfort.

## ***INTRODUCTION:***

Vibration is a critical element of automotive engineering analysis. If the oscillations created from a suspension's vibrations are too severe a bumpy and uncomfortable ride can occur, and in some cases massive damage can be inflicted to the vehicle. By analyzing vibration, specifically through examining suspension systems in modern automobiles, the optimal systems can be discovered.

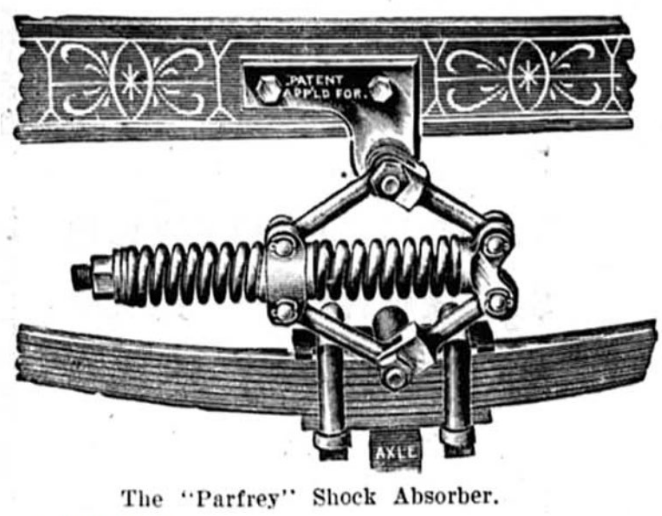
There are two kinds of suspension systems that will be discussed further in the report: dependent and independent. These systems will be compared with one another and the pros and cons will be weighed to determine which is better for what type of vehicle. Additionally, struts, coil overs, and system compactness will be examined.

Initially these elements will be discussed, followed by a mathematical analysis expanding on the explanations.

## ***DISCUSSION:***

Independent suspension is an automobile suspension system that allows each wheel on the same axle to move vertically, regardless of the other wheels. Most modern vehicles possess independent front suspension, and many vehicles have an independent rear suspension as well. Finally, a fully independent suspension has an independent suspension on each wheel. One very popular brand of strut used in independent suspension systems is the MacPherson strut. These suspension systems generally have better comfort and handling, as each wheel is unaffected by the others. Independent systems are most commonly used in smaller vehicles, as opposed to dependent suspension systems.

Dependent suspension systems consist of a solid axle that runs along the width of the frame. This allows the wheels on opposite sides to operate in unison. They are superior to independent suspension systems when driving on harsh terrain and are often found in large vehicles like SUVs and trucks.




Source | theoldmotor.com

Figure 1. The Parfrey Shock Absorber.

Early suspension systems like the one shown here utilized leaf springs, several layers of steel sheets shackled together to create a spring largely used in off roads and heavy duty applications because of its ability to suspend heavy loads, but are not known to provide a smooth ride.

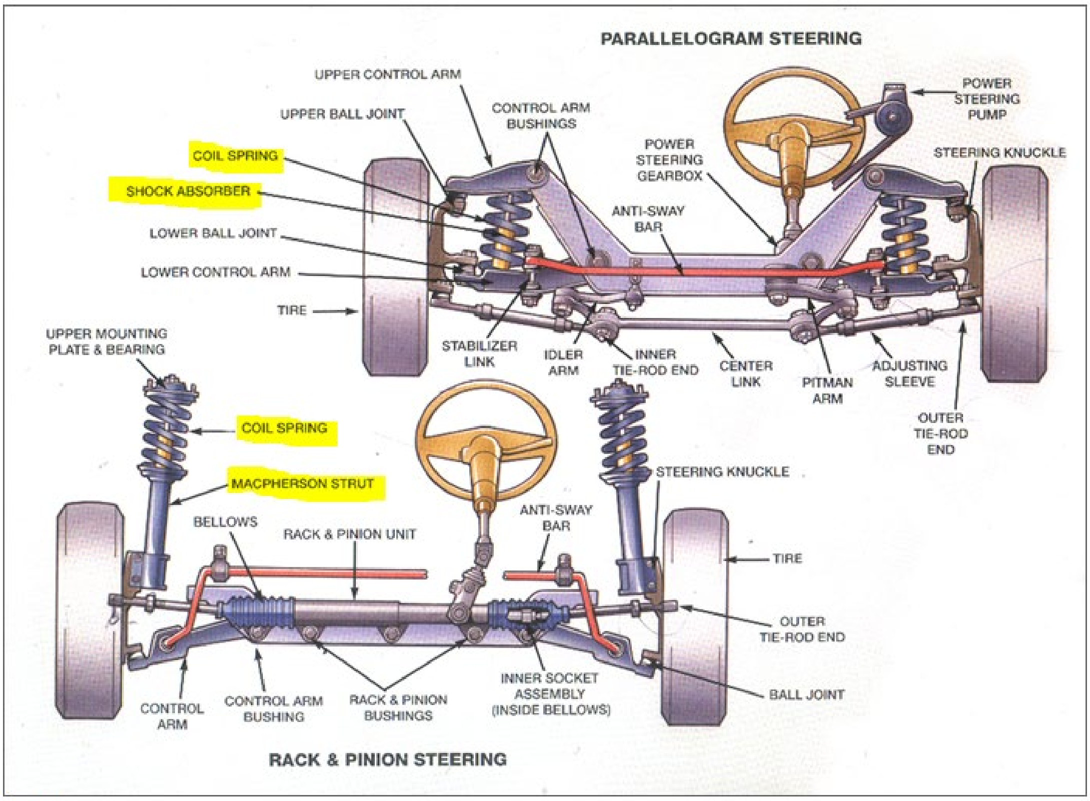



Figure 2. Springs, Shock Absorbers, and Struts

Modern automobiles however use shock absorbers or Macpherson struts which are basically the same thing but differ in its setup and are generally used on front wheel drive cars. Shock absorbers is a long tube shaped metal piston filled with gas, fluid or both and its job is to manage the recoil of the leaf springs to ensure a smoother ride by making sure the wheels do not jump off the road.

MacPherson struts like shocks is also a tube shaped metal piston with a built in coil spring so it eliminates the use of leaf springs and it is also a part of both the suspension and steering systems as it is partly involved in the movement of the wheels when the steering wheel is used.

***ANALYSIS:***

Quarter car model is used to determine mass displacement and mass acceleration. These effects are displayer in Figures!!!.

For any vehicle model, the general equation of motion is the following:

**[M]{a} + [C]{v} + [K]{x} = F**

[M] – Mass Matrix

[C] – Damping Coefficient

[K] – Stiffness Matrix

F - Force of road acting on vehicle

a – Acceleration

v – Velocity

x - Displacement

This equation is translated to the following when considering whether or not the equation for motion involves a sprung or unsprung mass:

$\mathbf{M*a_1 + K_s(x_1 - x_2) + C_s(v_1 - v_2) = 0}$

$\mathbf{m*a_2 + K_t(x_2 - w) + C_t(v_2 - dw/dt) – K_s(x_1 - x_2) - C_s(v_1 - v_2) = 0}$

M - Sprung Mass

m - Unsprung Mass

Ks - Suspension Stiffness

Kt - Tire Stiffness

Cs - Damping coefficient

Ct – Tire Damping Coefficient

w – Speed Bump Height

$X_1$ - Sprung Mass Movement (Vertical)

$X_2$ – Unsprung Mass Movement (Vertical)

Δ - Suspension Difference in length

- Formulas for natural frequencies for sprung and unsprang mass as follows:

$$\boldsymbol{\omega_n}(s) = \frac{1}{2\pi}\sqrt{\frac{(K_W * K_T)/(K_W + K_T)}{M_S}}$$

$\boldsymbol{\omega_n}(s) = Sprung\ mass\ Natural\ Frequency\ (Hz)$

$K_W$ = Wheel Rate

$K_T$ = Tire Spring Rate

$M_S$ = Sprung Mass

$$\boldsymbol{\omega_n}(us) = \frac{\mathbf{1}}{\mathbf{2\pi}}\sqrt{\frac{\boldsymbol{K_W + K_T}}{\boldsymbol{M_{US}}}}$$

$\boldsymbol{\omega_n}(us) = Unsprung\ mass\ Natural\ Frequency\ (Hz)$

$K_W$ = Wheel Rate

$K_T$ = Tire Spring Rate

$M_{US}$ = Unsprung Mass

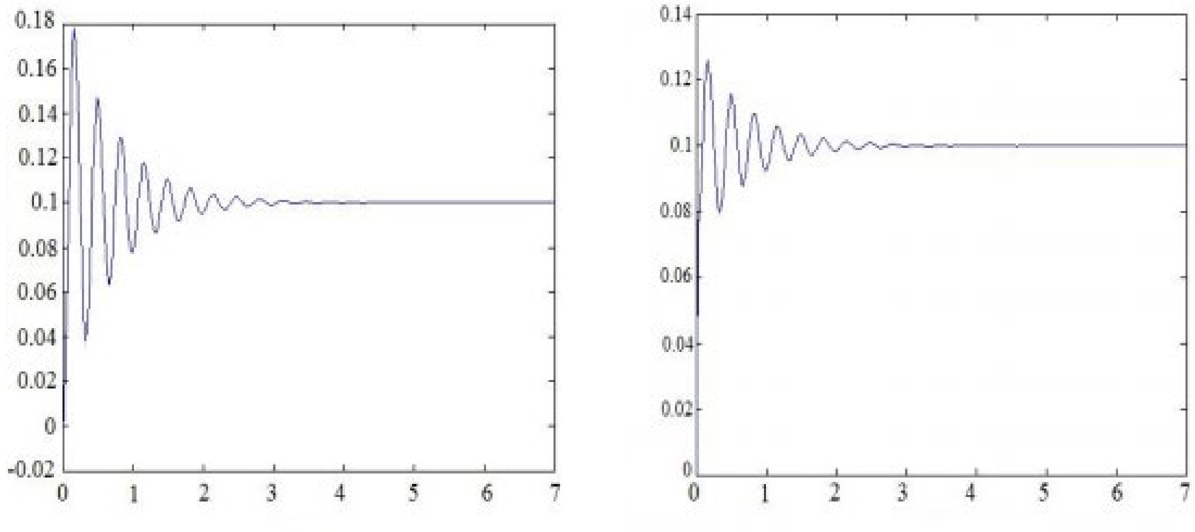



Figure 3. Sprung Mass Displacement vs Unsprang Mass Displacement. (Appendix)

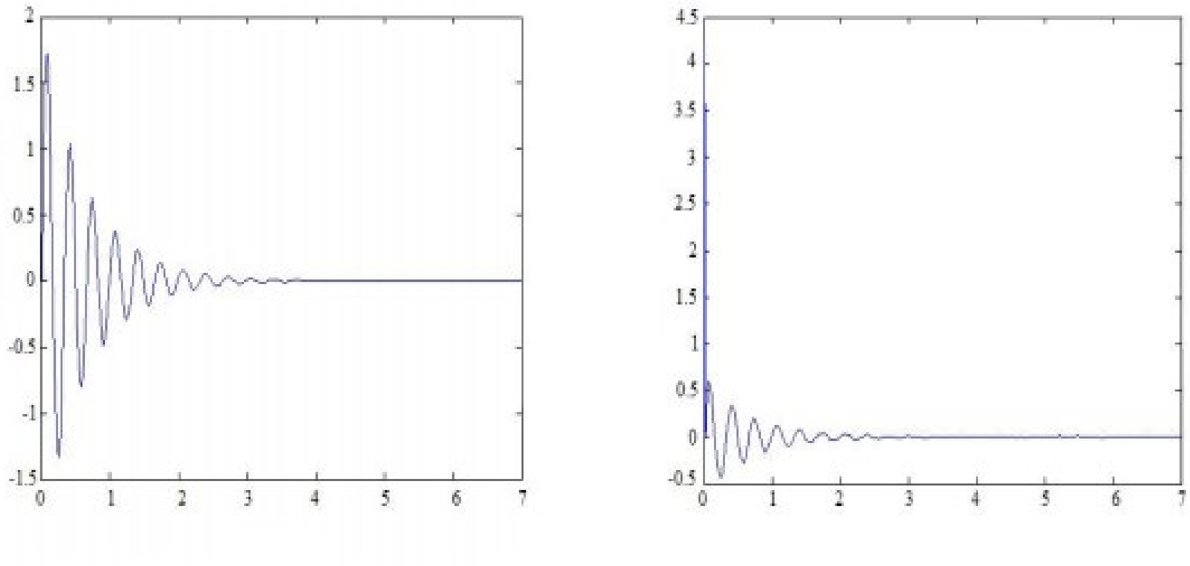



Figure 4. Sprung Mass Acceleration vs Unsprung Mass Acceleration. (Appendix)

***CONCLUSION:***

A brief analysis reveals that unsprung systems have less variation in vibration and return to their initial state sooner than their sprung counterparts.

It can be seen that as mass increases, natural frequency decreases, while as wheel rate and spring rates increase, natural frequency increases as well.

A more comfortable ride can be obtained through the process of eliminating displacement and acceleration spikes, which comes from determining the size of vehicle driven, what kind of load it carries, the road and weather conditions, the speed at which it travels, and many other conditions.

This report has been a small sample of potential impacts that certain vibrations can have on modern vehicles.

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MATLAB Help Function

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***APPENDIX:***

```
m=30; M=300; Cs=1100; Ct=3000; Ks=150000; Kt=310000; %Input variables

A=[0 0 0 1;0 0 1 -1;0 -(Ks/M) -(Cs/M) (Cs/M);-(Kt/m) (Ks/m) (Cs/m) (-(Cs+Ct)/m)]

B=[(Ct/m);-(Ct/m);((Ct*Cs)/(m*M));[(-(Cs*Ct)/(m*m))-((Ct*Ct)/(m*m))+(Kt/m)]]

C=[1 1 0 0; 0 0 1 0; 1 0 0 0; 0 0 0 1; 0 1 0 0 ] ;

D=[0]

t=0:0.01:10;

u=0.1*ones(size(t));

[Y,X]=lsim(A,B,C,D,u,t)

plot(t,Y)
```