3. Follow the appropriate sign convention: tension—positive; compression—negative; shear—clockwise rotation positive, counterclockwise rotation negative.
4. Plot σ_x and σ_y on the normal stress (horizontal) axis.
5. Plot the shear stress $\pm\tau$ associated with the normal stress for both the vertical face and the horizontal face. Label points X and Y.
6. Draw the XY line and Mohr's circle.
7. Determine the principal stresses and the maximum shear stress.
8. Determine $2\theta_{P1}$ and/or $2\theta_{P2}$ measuring from the X end of the reference line.
9. Determine $2\theta_s$.
10. Sketch two rotated elements, one for principal stresses and one for maximum shear stresses.

EXAMPLE 17.10 An element of a machine member is subjected to the stresses shown in Figure 17.32. Using Mohr's circle, (a) determine the magnitudes of the principal stresses and the maximum shear stresses, (b) determine the orientations of the principal planes and the planes on which the maximum shear stresses occur, and (c) show the results for the previous two parts on two rotated stressed elements.

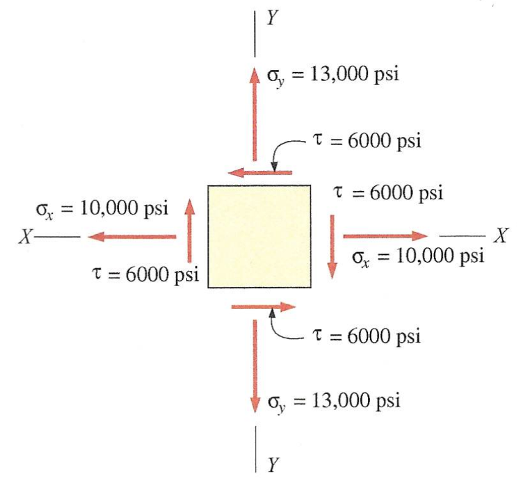


FIGURE 17.32 Stressed element.

Solution (a) In accordance with our sign convention, $\sigma_x = +10{,}000$ psi, $\sigma_y = +13{,}000$ psi, τ on the vertical face $= +6000$ psi, and τ on the horizontal face $= -6000$ psi. Mohr's circle is shown in Figure 17.33. Since it is not drawn to scale, mathematical relationships will be used along with the graphical construction.

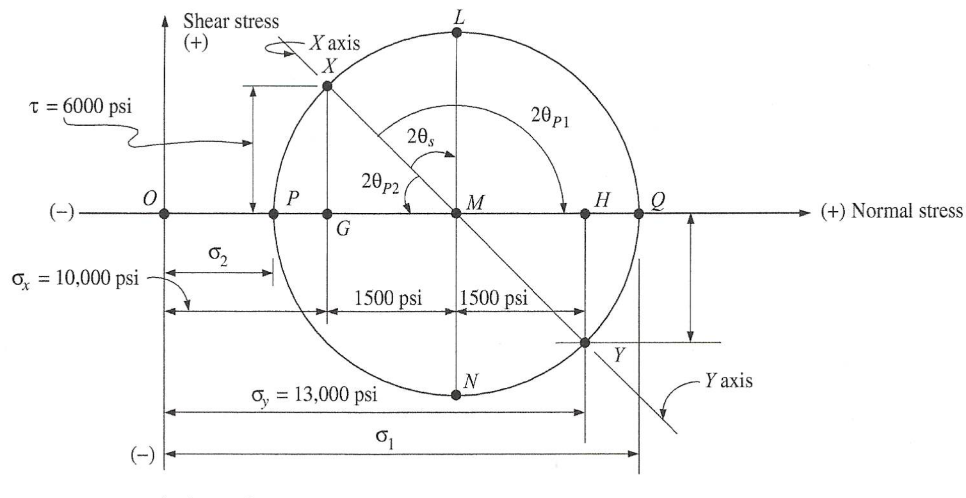


FIGURE 17.33 Mohr's circle.

First, plot $OG = +10{,}000$ psi (this is σ_x) and $OH = +13{,}000$ psi (this is σ_y) on the horizontal axis. Since the shear stress τ associated with σ_x is a positive value, draw a line perpendicular to the horizontal axis (upward) at G and plot point X so that $XG = +6000$ psi. Since τ associated with σ_y is a negative value, draw a line perpendicular to the horizontal axis (downward) at H and plot point Y so that $HY = -6000$ psi.

Then, draw the diameter XY and locate the point at which it crosses the horizontal axis M, which is the center of Mohr's circle. Compute the radius of the circle using triangle MHY:

$$GM = MH = \frac{\sigma_y - \sigma_x}{2} = \frac{13{,}000 - 10{,}000}{2} = 1500 \text{ psi}$$
$$MY = \sqrt{(MH)^2 + (HY)^2} = \sqrt{1500^2 + 6000^2} = 6185 \text{ psi}$$

Distances OQ and OP along the horizontal axis represent the maximum and minimum principal stresses σ_1 and σ_2:

$$\sigma_1 = OM + MQ = (10{,}000 + 1500) + 6185 = +17{,}685 \text{ psi (tension)}$$
$$\sigma_2 = OM - MP = (10{,}000 + 1500) - 6185 = +5315 \text{ psi (tension)}$$

The maximum shear stress τ' is represented by point L (or point N). The magnitude of the maximum shear stress is equal to the radius of the circle, 6185 psi. Note also that the normal stresses acting on the planes of maximum shear stress can be determined at points L and N. Also, compare with Equation (17.12). Thus,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} = \frac{10{,}000 + 13{,}000}{2} = +11{,}500 \text{ psi (tension)}$$

(b) The inclination of the principal stresses is determined by measuring from the X end of reference axis X–Y to the maximum principal stress at point Q. The clockwise angle of $2\theta_{P1}$ is labeled on the circle. From triangle XMG,

$$2\theta_{P2} = \angle XMG = \tan^{-1}\frac{XG}{GM} = \tan^{-1}\frac{6000}{1500} = 4.0 = 75.96°$$
$$2\theta_{P1} = 180° - 75.96° = 104.04°$$
$$\theta_{P1} = 52.0° \text{ (clockwise)}$$

Therefore, the maximum principal stress f_1 is inclined at a clockwise angle of 52.0° with the X–X axis of the original stressed element.

The inclination to the plane of maximum positive shear stress is represented by θ_s. Point L on the circle represents this stress and lies clockwise $2\theta_s$ from the X end of the X–Y reference line. The simplest way to obtain this angle is to subtract 90° from $2\theta_{P1}$:

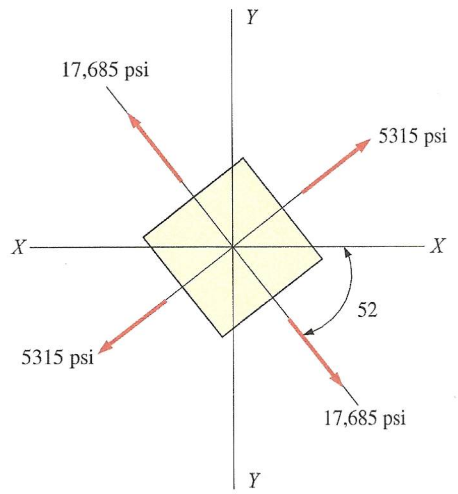
$$2\theta_s = 2\theta_{P1} - 90°$$

from which

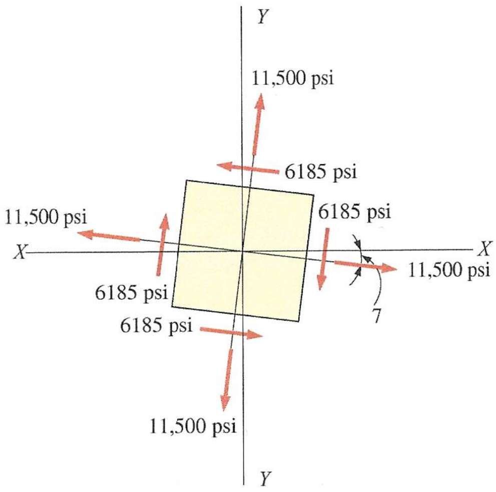
$$\theta_s = \theta_{P1} - 45° = 7.0° \text{(clockwise)}$$

Since angle $2\theta_s$ is shown as clockwise to the maximum positive shear stress (radius ML), the element is shown rotated clockwise 7° in Figure 17.34b. Since the shear stress at point L is positive, it is shown acting downward on the face of the element that corresponds to the right vertical face of the original stressed element. This action tends to rotate the element clockwise and is consistent with the previously discussed sign convention. Note that on the stressed element the planes of maximum shear stress are oriented at 45° to the principal planes.

(c) Results from parts (a) and (b) are shown graphically in Figure 17.34.



(a) Principal stress element



(b) Maximum shear stress element

FIGURE 17.34 Results for Example 17.10.

SUMMARY BY SECTION NUMBER

17.2 Biaxial bending occurs where a beam is subjected to a load or loads that produce bending about both the strong and the weak axes. For the simple case where no torsion is produced, the combined stresses of interest can be determined from

$$f = \pm\frac{M_x}{S_x} \pm \frac{M_y}{S_y} \qquad \textbf{(17.1)}$$

17.3 When a member of relatively short span length is subjected to both transverse loads and direct axial loads, bending stresses and axial stresses are developed. The combined stress at any location is a simple algebraic sum of the two types of stress and is expressed as

$$f = \pm\frac{P}{A} \pm \frac{Mc}{I} \qquad \textbf{(17.2)}$$

17.4 When a short compression member is subjected to a load that lies on only one of the centroidal axes, bending stresses and axial stresses are developed. If the moment is expressed in terms of the load and eccentricity ($M = Pe$), the combined stress expression is

$$f = -\frac{P}{A} \pm \frac{Pec}{I} \qquad \textbf{(17.3)}$$

17.5 When a short compression member is subjected to an eccentric load, combined stresses (either tension or compression) will develop at the outer edges of the cross section. If the bending stress is equal to the axial stress, the stress at one outer edge will be zero. The maximum load eccentricity for which this zero stress will develop in a rectangular shape is

$$e = \frac{d}{6} \qquad \textbf{(17.4)}$$

where d represents the dimension parallel to the eccentricity.

17.6 If a load applied to a short compression member is eccentric to both centroidal axes, the member is subjected to a double eccentricity, and the combined stress expression is

$$f = -\frac{P}{A} \pm \frac{Pe_1c_1}{I_y} \pm \frac{Pe_2c_2}{I_x} \qquad \textbf{(17.5)}$$

17.7 The combination of normal and shear stress results in maximum and minimum normal and shear stresses being developed on planes that are inclined with respect to the planes of stress being combined. The planes on which the normal stresses become maximum or minimum are called *principal planes*. The normal stresses acting on the planes are called *principal stresses* and can be obtained from

$$f_{1,2} = \left(\frac{f_x + f_y}{2}\right) \pm \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2} \qquad \textbf{(17.9)}$$

Shear stresses on the principal planes are equal to zero. The maximum shear stress developed on an inclined plane is

$$f'_{s\,(\max)} = \pm\sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2} \qquad \textbf{(17.11)}$$

17.8 and **17.9** Mohr's circle is a graphical (or semigraphical) alternate method of calculating normal stresses and shear stresses acting on any inclined plane at any point in a stressed body. It can be used both for uniaxial and for biaxial loading conditions.

PROBLEMS

Unless noted otherwise, neglect the weights of the members in these problems.

Section 17.2 Biaxial Bending

17.1 A 100-mm-by-100-mm (S4S) timber beam on a simple span of 4 m is subjected to vertical and lateral point loads placed at midspan, as shown. The lines of action of the loads pass through