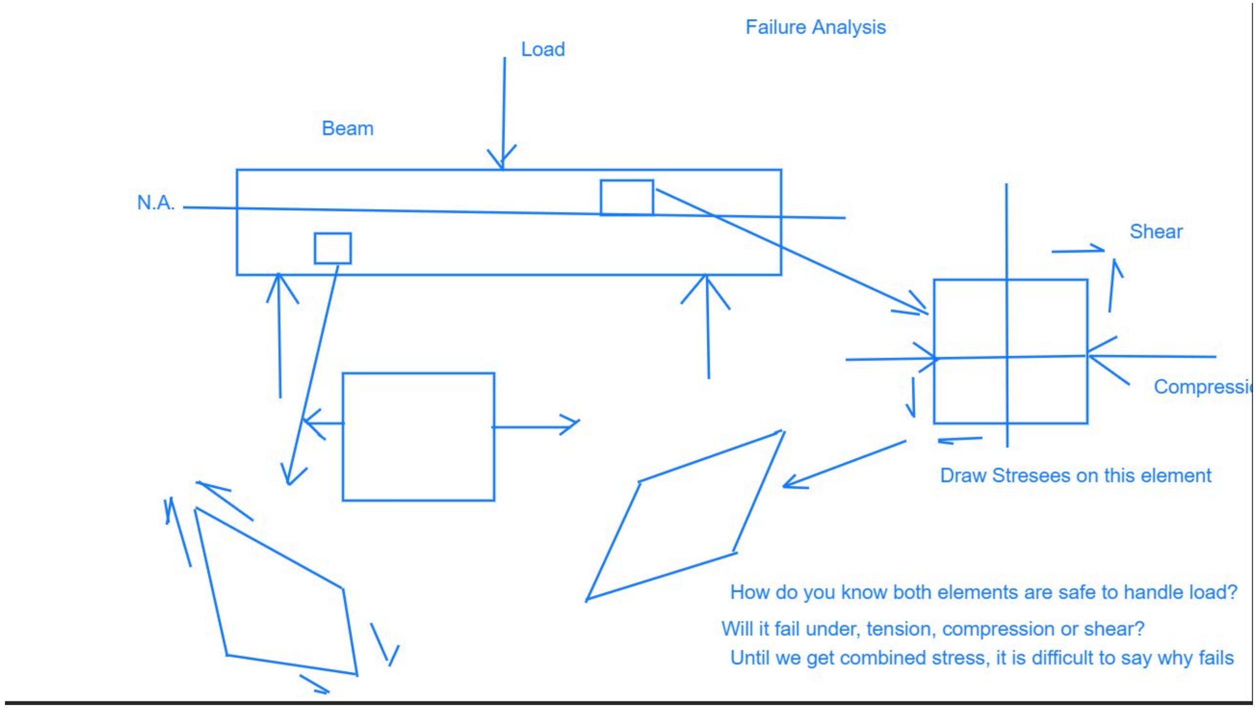
Combined stress

MECH 2333

Combined normal and shear stress



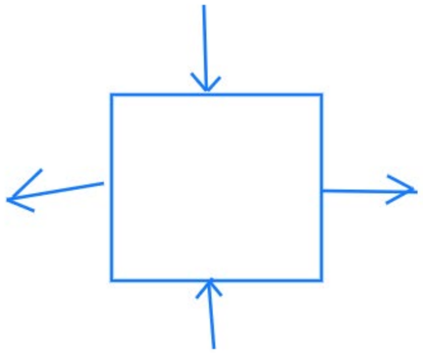
If stress in x dir +50 MPa

Stress in y dir -30 MPa

Shear stress 10 MPa

Determine stress acting on an element oriented 30 deg clockwise w.r.t original element

clockwise dir is negative...



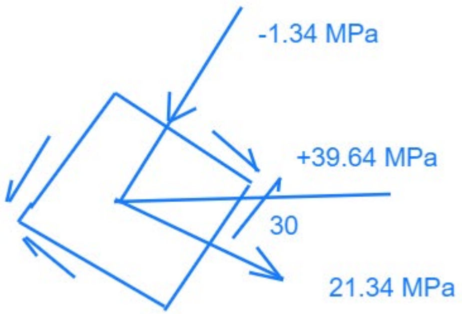
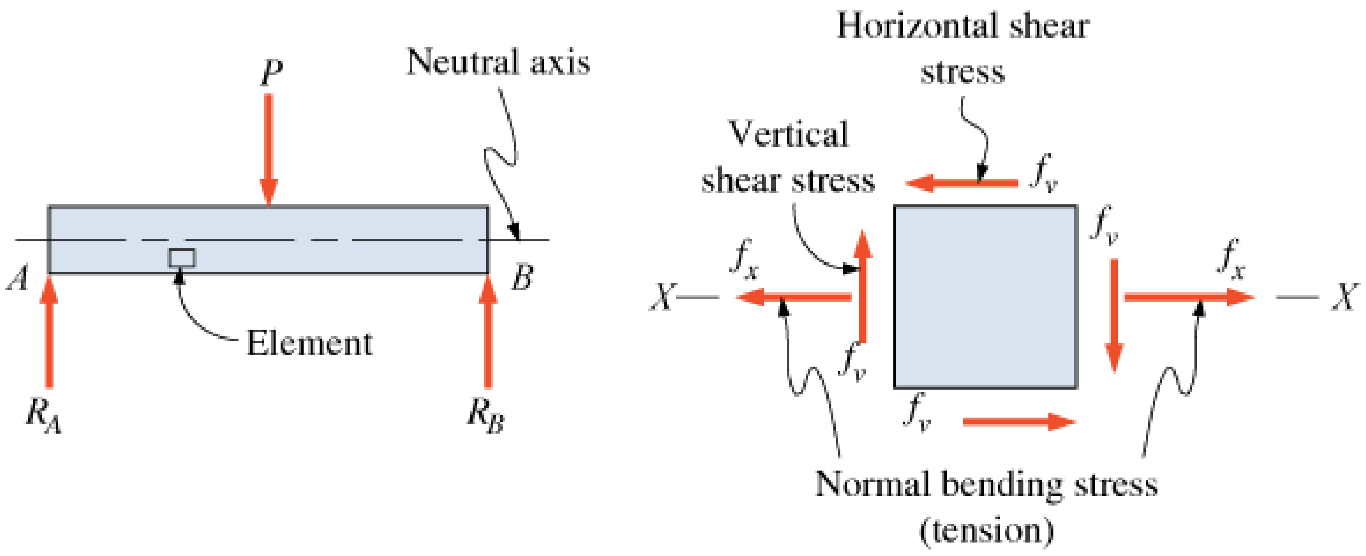


Figure 17.18 Combined normal and shear stress.



(a) Loaded beam (b) Magnified stress element

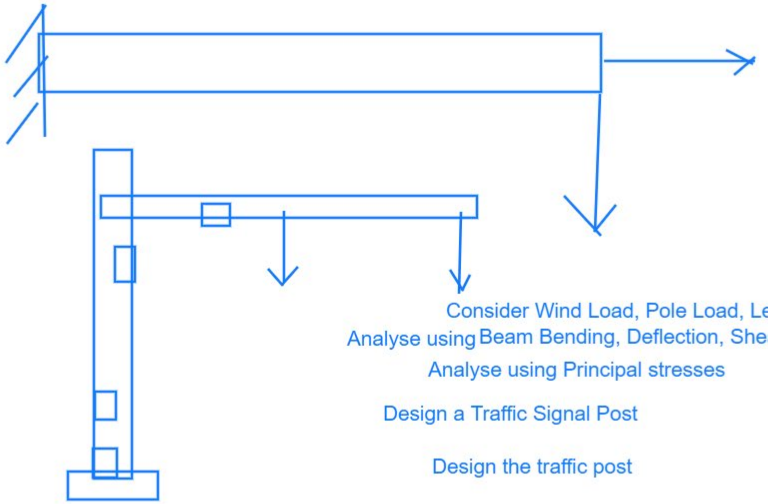
Design a Traffic Post
Considering the following conditions-

Make a group of 3

six groups

Maximum wind load, Post load, Beam load

Consider Severe wind direction



Project credit, 10 -15%

Consider Wind Load, Pole Load, Length of the post,

Analyse using Beam Bending, Deflection, Shear Stress

Analyse using Principal stresses

Design a Traffic Signal Post

Design the traffic post

Mohr Circle for Plane Stress

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi \tag{3–8}$$

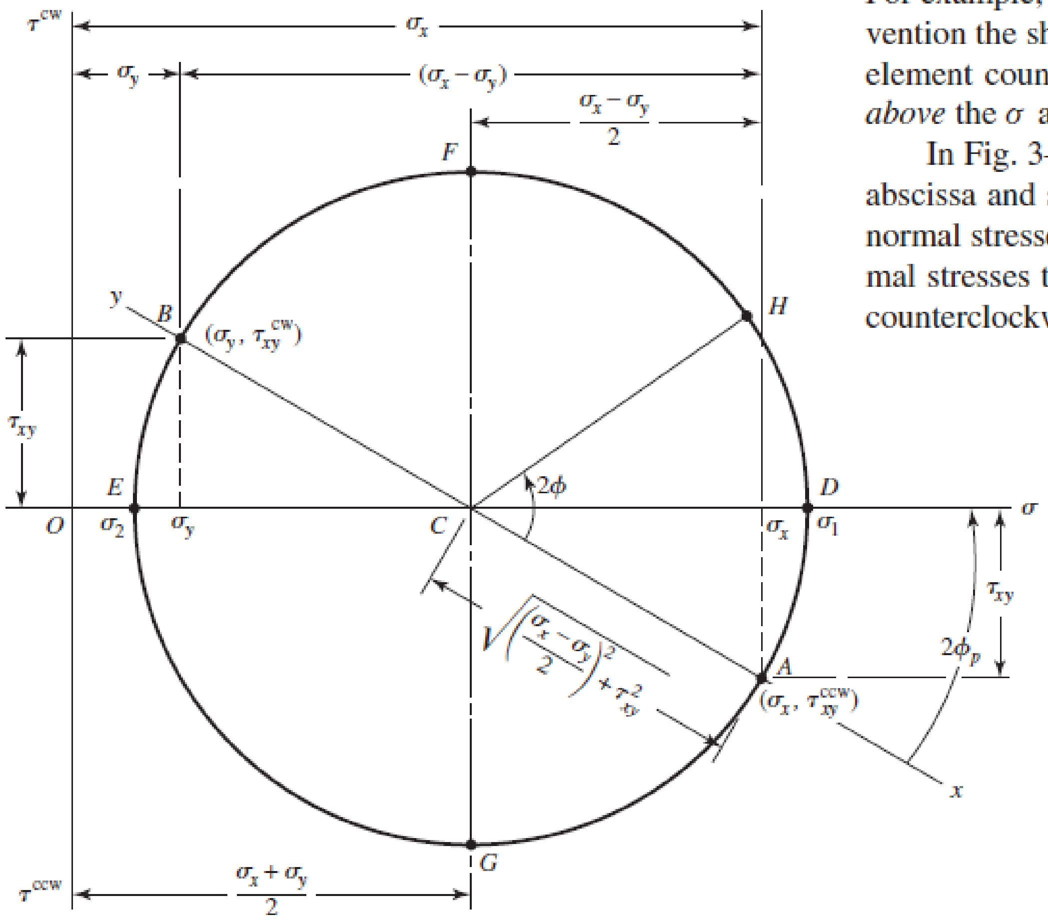
$$\tau = -\frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi \tag{3–9}$$

Equations (3–8) and (3–9) are called the *plane-stress transformation equations*.

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{3–14}$$

Figure 3–10

Mohr's circle diagram.



Mohr's Circle Shear Convention

This convention is followed in drawing Mohr's circle:

- Shear stresses tending to rotate the element clockwise (cw) are plotted *above* the σ axis.
- Shear stresses tending to rotate the element counterclockwise (ccw) are plotted *below* the σ axis.

For example, consider the right face of the element in Fig. 3–8*b*. By Mohr's circle convention the shear stress shown is plotted *below* the σ axis because it tends to rotate the element counterclockwise. The shear stress on the top face of the element is plotted *above* the σ axis because it tends to rotate the element clockwise.

In Fig. 3–10 we create a coordinate system with normal stresses plotted along the abscissa and shear stresses plotted as the ordinates. On the abscissa, tensile (positive) normal stresses are plotted to the right of the origin O and compressive (negative) normal stresses to the left. On the ordinate, clockwise (cw) shear stresses are plotted up; counterclockwise (ccw) shear stresses are plotted down.

Principal Stress from applied stress

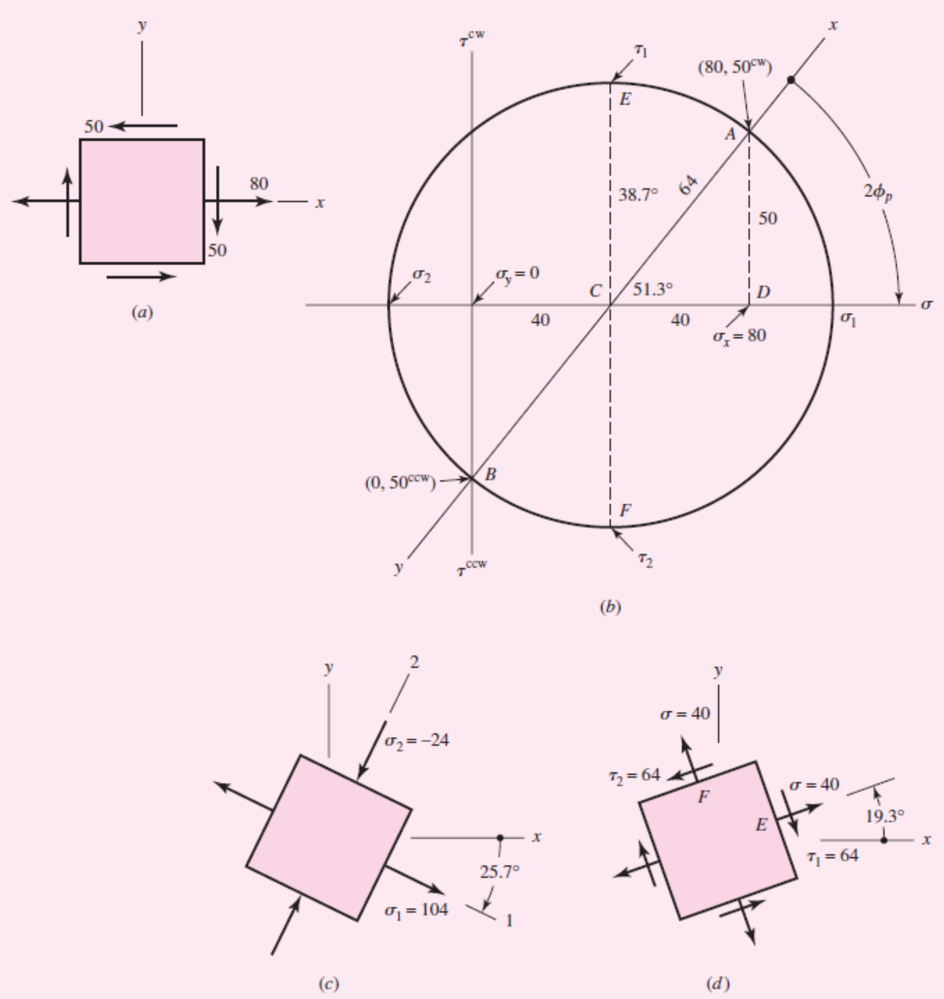
Sigma 1,2 = (Sigmax+Sigmay)/2±R

Tau1,2 = ±R

A stress element has $\sigma_x = 80$ MPa and $\tau_{xy} = 50$ MPa cw, as shown in Fig. 3–11*a*.

(*a*) Using Mohr's circle, find the principal stresses and directions, and show these on a stress element correctly aligned with respect to the xy coordinates. Draw another stress element to show τ_1 and τ_2, find the corresponding normal stresses, and label the drawing completely.

(*b*) Repeat part *a* using the transformation equations only.



(*b*) The transformation equations are programmable. From Eq. (3–10),

$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-50)}{80}\right) = -25.7^\circ, 64.3^\circ$$

From Eq. (3–8), for the first angle $\phi_p = -25.7^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(-25.7)] + (-50)\sin[2(-25.7)] = 104.03 \text{ MPa}$$

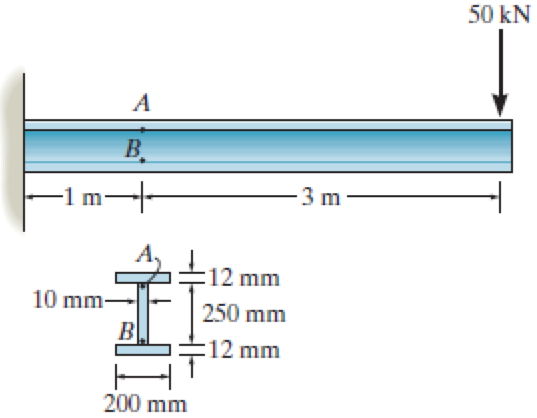
The shear on this surface is obtained from Eq. (3–9) as

$$\tau = -\frac{80-0}{2}\sin[2(-25.7)] + (-50)\cos[2(-25.7)] = 0 \text{ MPa}$$

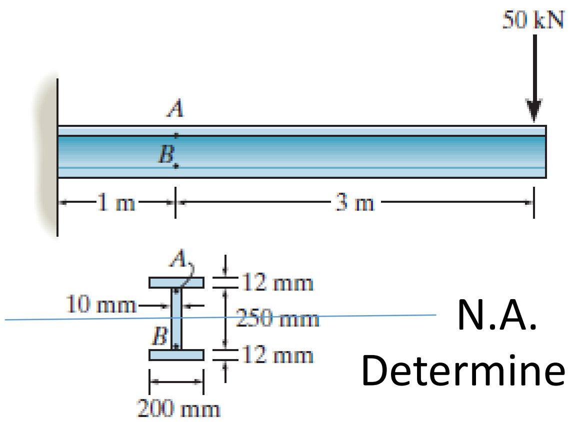
which confirms that 104.03 MPa is a principal stress. From Eq. (3–8), for $\phi_p = 64.3^\circ$,

$$\sigma = \frac{80+0}{2} + \frac{80-0}{2}\cos[2(64.3)] + (-50)\sin[2(64.3)] = -24.03 \text{ MPa}$$

The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.



Solution



N.A.

Determine I of the section

SOLUTION

$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

Q= A*y= Area moment

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

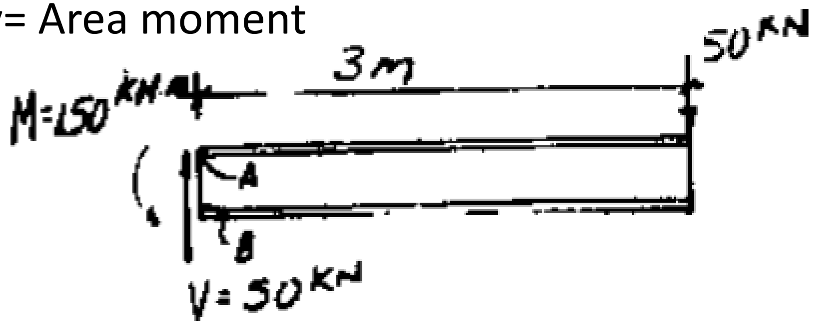
$$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$$

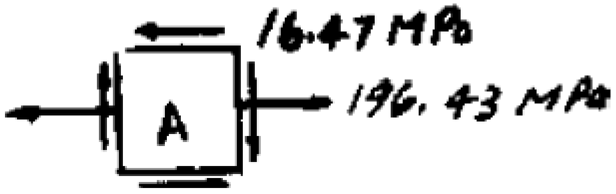
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$$\sigma_1 = 198\ \text{MPa}$$

$$\sigma_2 = -1.37\ \text{MPa}$$





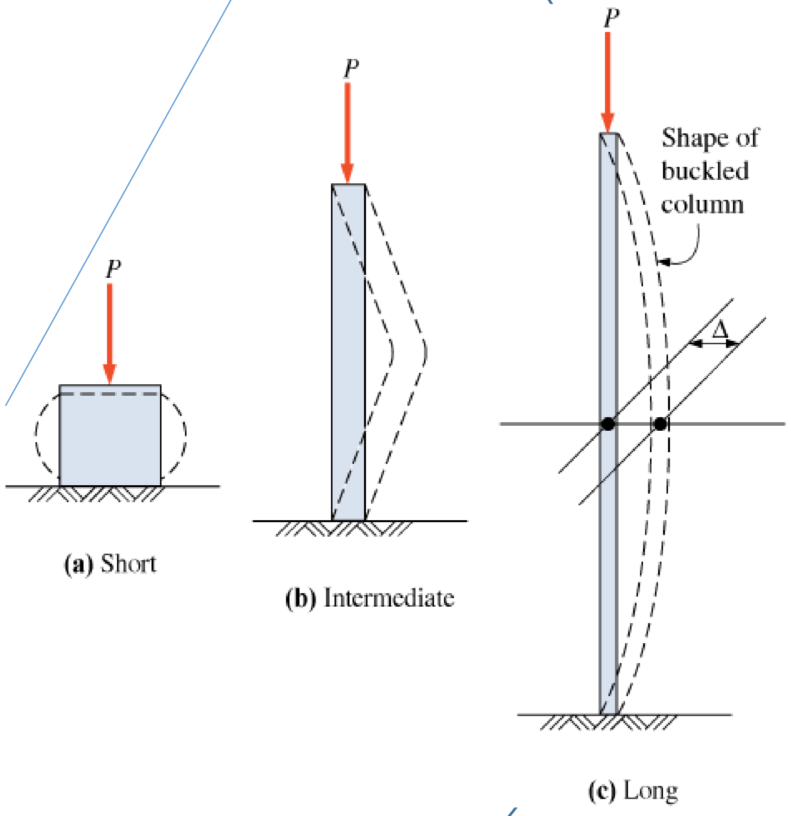
Shear, V=50 KN

Moment, M=150 KN.M

Lets find principle stress using mohr's Circle

Also Solve for Point B

Column



Critical Load= How much load the column can take before Buckling?

$$P_e = \frac{\pi^2 EI}{L^2} \tag{18.1}$$

where P_e = the critical load; the concentric load that will cause initial buckling (lb) (N)
π = a mathematical constant (3.1416)
E = the modulus of elasticity of the material (psi) (MPa)
I = the least moment of inertia of the cross section (in.^4) (mm^4)
L = the length of the column from pin end to pin end (in.) (mm)

Slenderness ratio, l/r, determines if the beam is long or intermediate kips.

Tests have verified that Euler's formula accurately predicts the buckling load if conditions are such that the buckling stress is less than the proportional limit of the material and adherence to the basic assumptions is maintained. Since the buckling stress must be compared with the proportional limit, Euler's formula is commonly written in terms of stress, which can be derived from the preceding buckling load formula using the relationship $r = \sqrt{I/A}$ or $I = Ar^2$ (see Section 8.5):

$$f_e = \frac{\pi^2 E}{(L/r)^2} \qquad \textbf{(18.2)}$$

where f_e = the critical stress; the uniform compressive stress at which initial buckling occurs (same units as E)

r = the least radius of gyration of the cross section (same units as L)

EXAMPLE 18.1 A 25-mm-diameter steel rod, shown in Figure 18.2, is used as a pin-connected compression member. Calculate the critical load using Euler's formula. The proportional limit for the steel is 235 MPa and the modulus of elasticity is 207×10^3 MPa. Assume the length of the rod to be (a) 1 m and (b) 2 m.

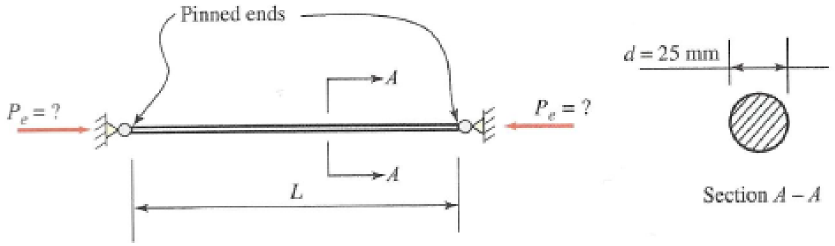


FIGURE 18.2 Pin-connected compression member.

Solution Refer to Table 3 inside the back cover for properties of areas:

$$A = 0.7854d^2 = 0.7854(25 \text{ mm})^2 = 491 \text{ mm}^2$$

$$r = \frac{d}{4} = \frac{25 \text{ mm}}{4} = 6.25 \text{ mm}$$

(a) For a 1-m length of rod,

$$f_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 \, (207 \times 10^3 \text{ MPa})}{[(1000 \text{ mm})/(6.25 \text{ mm})]^2}$$

$$= 79.8 \text{ MPa} < 235 \text{ MPa} \qquad \textbf{(O.K.)}$$

The critical load can then be calculated from

$$P_e = f_e A = (79.8 \text{ MPa})(491 \text{ mm}^2) = 39.2 \times 10^3 \text{ N} = 39.2 \text{ kN}$$

(b) For a 2-m length of rod,

$$f_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 \, (207 \times 10^3 \text{ MPa})}{[(2000 \text{ mm})/(6.25 \text{ mm})]^2}$$

$$= 19.95 \text{ MPa} < 235 \text{ MPa} \qquad \textbf{(O.K.)}$$

$$P_e = f_e A = (19.95 \text{ MPa})(491 \text{ mm}^2) = 9.80 \times 10^3 \text{ N} = 9.8 \text{ kN}$$

r=sqrt(I/A)
= d/4

Proportional limit=Yield Stress

Solution From Table 3,

$$A = 0.7854d^2 = 0.7854(1 \text{ in.})^2 = 0.7854 \text{ in.}^2$$

$$r = \frac{d}{4} = \frac{1.0 \text{ in.}}{4} = 0.25 \text{ in.}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi(1 \text{ in.})^4}{64} = 0.0491 \text{ in.}^4$$

(a) The shortest length L for which Euler's formula applies will be that length for which the critical stress is equal to the proportional limit:

$$f_e = \frac{\pi^2 E}{(L/r)^2}$$

Solve for L:

$$L^2 = \frac{\pi^2 E r^2}{f_e} = \frac{\pi^2 (30{,}000{,}000 \text{ psi})(0.25 \text{ in.})^2}{50{,}000 \text{ psi}} = 370.1 \text{ in.}^2$$

from which

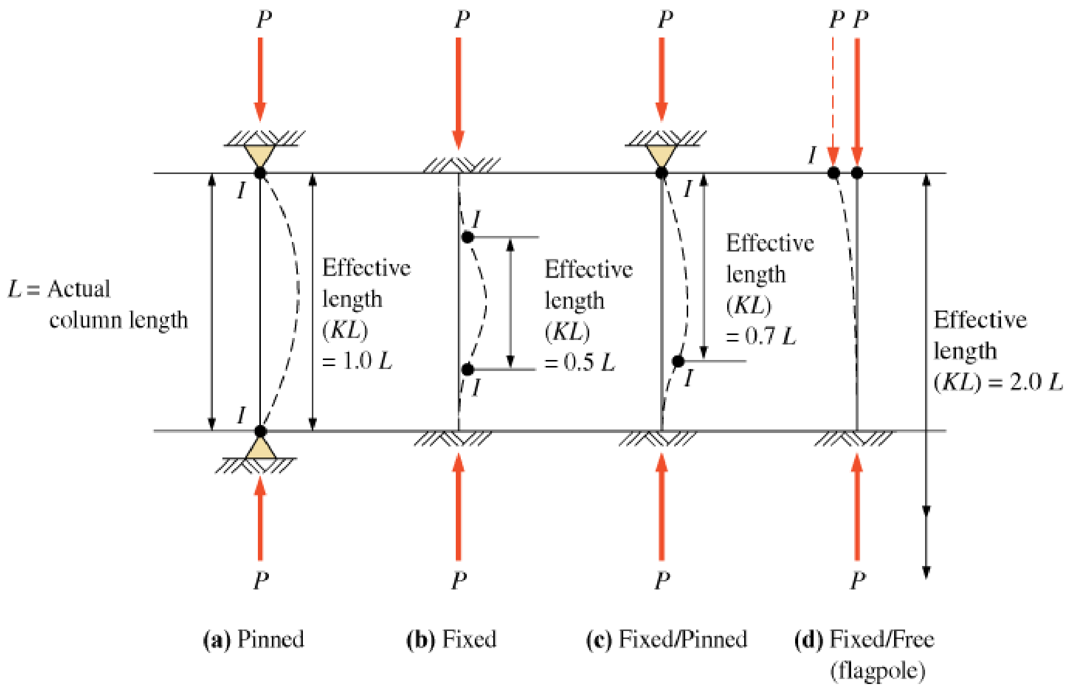
$$L = 19.24 \text{ in.}$$

(b) If $L = 48$ in., Euler's formula is valid (48 in. > 19.24 in.). Therefore, the critical load can be calculated from Equation (18.1):

$$P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (30{,}000{,}000 \text{ psi})(0.0491 \text{ in.}^4)}{(48 \text{ in.})^2} = 6310 \text{ lb}$$

Effective length factor

Effective length depends on the end condition



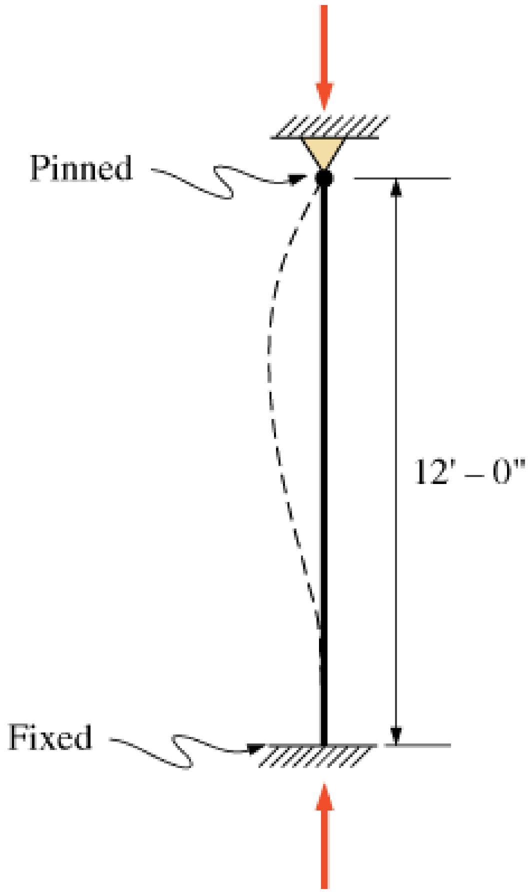
Note: *I* indicates inflection points

TABLE 18.1 Effective length factors

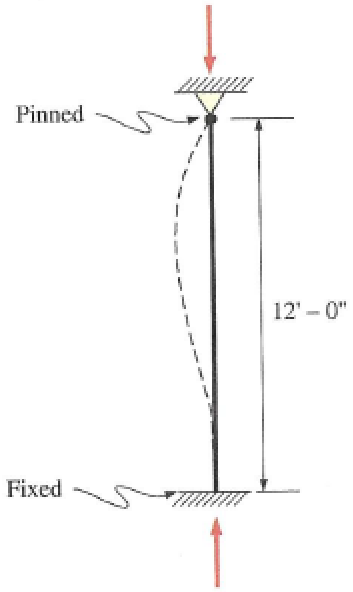
Idealized End Conditions	Theoretical K Value	Recommended K Value for Design and Analysis	Number of Times Stronger Than Pinned-End Column[a]
Pinned	1.0	1.0	1
Fixed	0.5	0.65	4
Fixed/pinned	0.7	0.80	2
Fixed/free	2.0	2.10	$\frac{1}{4}$

[a]Using the theoretical K value.

EXAMPLE 18.3 A column is composed of a 3-in.-diameter standard-weight steel pipe (Pipe 3 Std.) and is solidly embedded in a concrete foundation at its base and pinned at the top. The column is 12 ft long, as shown in Figure 18.5. Assuming the proportional limit to be 34 ksi, find the buckling load using the Euler theory.



Building Column
Of Structural column
Has both fixed ends



If both ends fixed, K =.65,
Will load capacity go up or down?
By how much? What percentage?

FIGURE 18.5 Column for Example 18.3.

Solution This is a fixed-pinned column. From Appendix B, the cross-sectional properties for this pipe are $A = 2.23 \text{ in.}^2$ and $r = 1.16$ in.

From Table 18.1, select $K = 0.8$ (recommended K for design/analysis). Calculate f_e using Equation (18.4):

$$f_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2(30{,}000 \text{ k/in.}^2)}{\left(\frac{0.8(144 \text{ in.})}{1.16 \text{ in.}}\right)^2} = 30.0 \text{ k/in.}^2$$

Note that 30 ksi < 34 ksi, therefore the Euler theory is applicable.

$$P_e = f_e A = 30.0 \text{ k/in.}^2 \,(2.23 \text{ in.}^2) = 66.9 \text{ k}$$

That means, this column can carry 67 Kilo Pound Load