Pie Challenge - Mathematical Induction

List the members of your team:

For **5 points** on Exam #4, submit a correct solution to the following challenge. Once you are satistifed you have the correct solution, divide your pie accordingly and enjoy!
n people are at a party. Use **mathematical induction** to show that n people can divide a pie (where each person gets one or more separate pieces of the pie) so that the pie is divided fairly, that is, in the sense that each person thinks he or she got at least $(1/n)$th of the pie.

In writing up your solution, make sure you identify your **basis step** (start with $n = 2$ people), your **inductive hypothesis** and explain your **inductive step**.

Solution:

For the **basis step** ($n = 2$) the first person cuts the cake into two portions that she thinks are each 1/2of the pie, and the second person chooses the portion he thinks is at least 1/2 of the pie (at least one of the pieces must satisfy that condition).

For the **inductive step**, suppose there are $k+1$ people. By the **inductive hypothesis**, we can suppose that the first k people have divided the pie among themselves so that each person is satisfied that he got at least a fraction $1/k$ of the pie. Each of them now cuts his or her piece into $k+1$ pieces of equal size. The last person gets to choose one piece from each of the first k people's portions. After this is done, each of the first k people is satisfied that she still has

$$\frac{1}{k}\frac{k}{(k+1)} = \frac{1}{(k+1)}$$

of the pie! To see that the last person is satisfied, suppose that he thought that the ith person ($1 \leq i \leq k$) had a portion p_i of the pie, where$\sum_{i=1}^{k} p_i = 1$. By choosing what he thinks is the largest piece from each person, he is satisfied that he has at least

$$\frac{\sum_{i=1}^{k} p_i}{(k+1)} = \frac{1}{(k+1)}\sum_{i=1}^{k} p_i = \frac{1}{(k+1)}$$

of the pie.