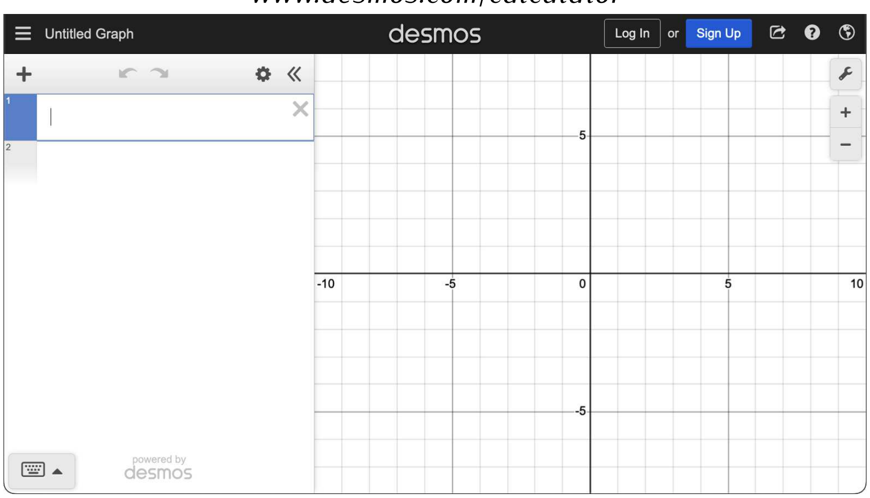
Session 4

Introduction to Desmos

4.1 Basics of the Desmos Graphing Calculator

We now give a short introduction to the Desmos Graphing Calculator which can be used to graph functions such as the ones discussed in the previous chapter. The Desmos Graphing Calculator can be found at the URL:

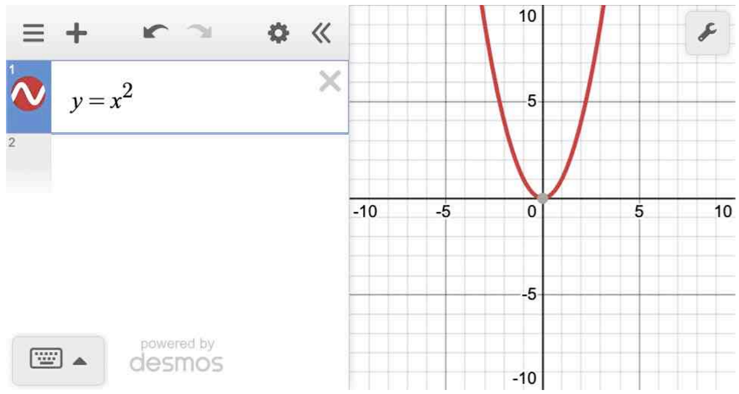
www.desmos.com/calculator



It is worth noting that there is also a mobile app (both for iOS and for android) with the same functionality.

Demos mobile app icon:

It is straightforward to graph a function in Desmos, say $y = x^2$.



Moreover, we can easily locate local maxima and minima of a function (the peaks and valleys of its graph), as well as its x- and y-intercepts by simply clicking on the graph. The x-intercepts are also commonly called **zeros** or **roots** of the function. In other words, a root or a zero of a function f is a number x for which $f(x) = 0$.

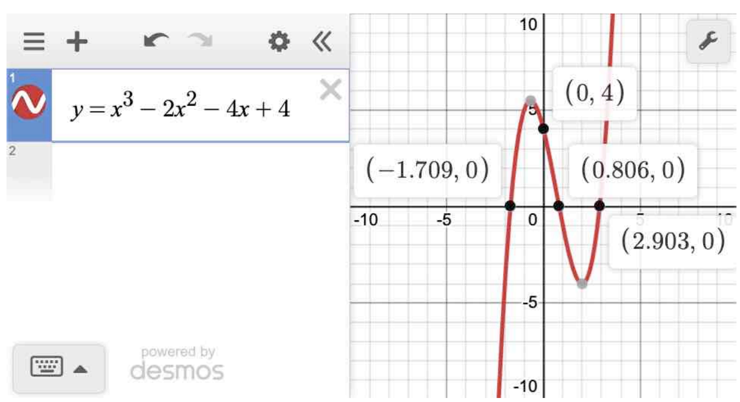
Example 4.1. Graph the function $y = x^3 - 2x^2 - 4x + 4$.

a) Approximate the x-intercepts and the y-intercept of the function.

b) Approximate the (local) maximum and minimum. (A (local) maximum or minimum is also called an **(local) extremum**.)

Solution.

a) Enter the function in Desmos and click on the x-intercepts (points where the graph intersects with the x-axis) and the y-intercept (the point where the graph intersects with the y-axis).

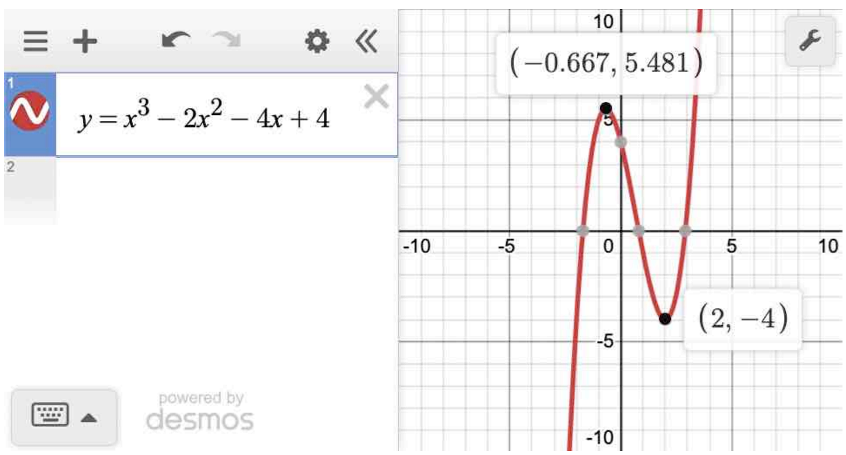


There are three x-intercepts and one y-intercept:

$$\begin{aligned} x\text{-intercepts: } & (x,y) \approx (-1.709, 0), \quad (x,y) \approx (0.806, 0) \\ & (x,y) \approx (2.903, 0) \\ y\text{-intercept: } & (x,y) = (0, 4) \end{aligned}$$

Note that the y-intercept is precisely at $(0, 4)$, whereas the x-intercepts are only approximated.

b) Similarly, we obtain the local maximum and the local minimum.



$$\text{local max: } (x,y) \approx (-0.667, 5.481), \qquad \text{local min: } (x,y) = (2, -4)$$

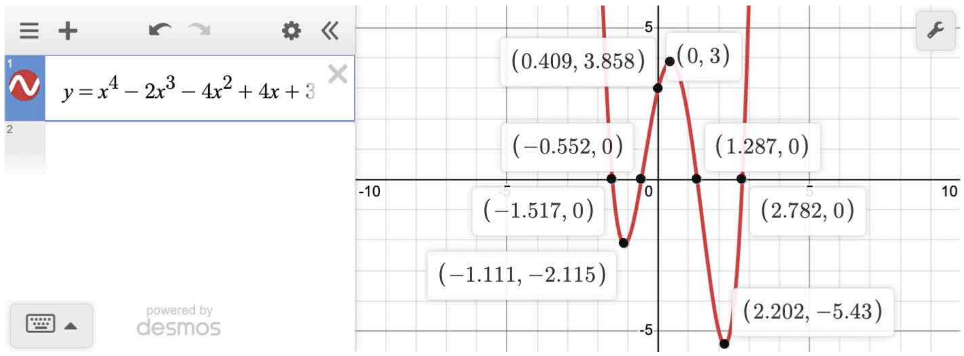
One can check (using methods from calculus) that the local minimum is precisely at $(2, -4)$, whereas the local maximum is only approximated. $\square$

Example 4.2. For the two functions below, find all intercepts and all extrema. Approximate your answer to the nearest thousandth.

a) $f(x) = x^4 - 2x^3 - 4x^2 + 4x + 3$ b) $f(x) = x^3 - 9x^2 - x$

Solution.

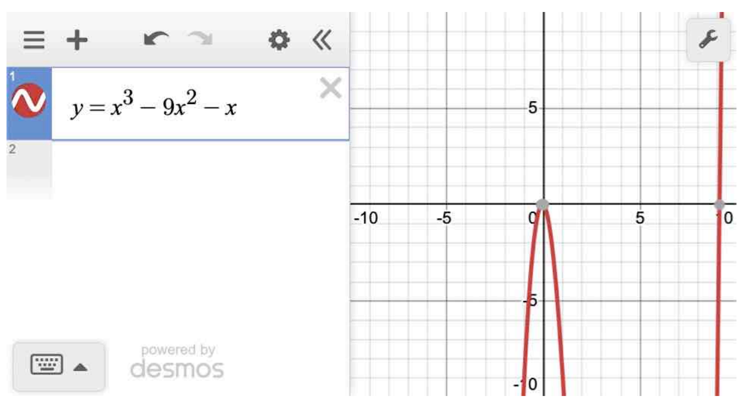
a) Graphing the function in Desmos, we can read off the coordinates of the wanted points (the intercepts and extrema) by clicking on them.



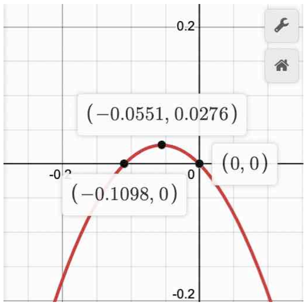
$$\begin{aligned}
x\text{-intercepts: } & (x,y) \approx (-1.517, 0), \quad (x,y) \approx (-0.552, 0), \\
& (x,y) \approx (1.287, 0), \quad (x,y) \approx (2.782, 0) \\
y\text{-intercept: } & (x,y) = (0,3) \\
\text{local maximum: } & (x,y) \approx (0.409, 3.858) \\
\text{local mininima: } & (x,y) \approx (-1.111, -2.115), \quad (x,y) \approx (2.202, -5.430)
\end{aligned}$$

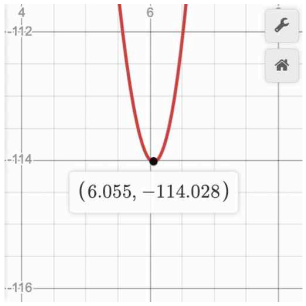
Here, Desmos already rounded coordinates to the nearest thousandth. For example, the maximum with one more digit is $(x,y) \approx (0.4088, 3.8580)$, which rounds to $(0.409, 3.858)$.

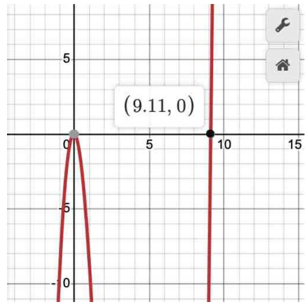
b) Graphing $f(x) = x^3 - 9x^2 - x$ shows that we don't have a complete view of all the points of interest in the initial viewing window.



Zooming in and out (by using the plus and minus symbols on the graph or by scrolling on the screen), we can get a closer view of the wanted points.







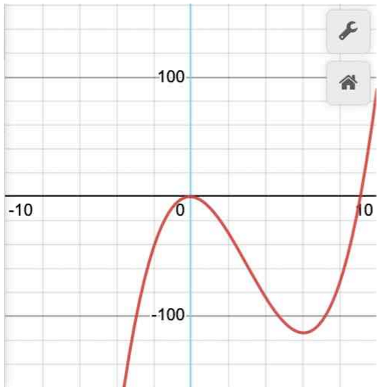
Note that by zooming in, Desmos may display more than three digits after the decimal point. Rounding to the nearest thousandth gives the following answers.

$$
\begin{aligned}
x\text{-intercepts: } & (x,y) \approx (-0.110, 0), \quad (x,y) = (0,0), \\
& (x,y) \approx (9.110, 0) \\
y\text{-intercept: } & (x,y) = (0,0) \\
\text{local maximum: } & (x,y) \approx (-0.055, 0.028) \\
\text{local mininimum: } & (x,y) \approx (6.055, -114.028)
\end{aligned}
$$

□

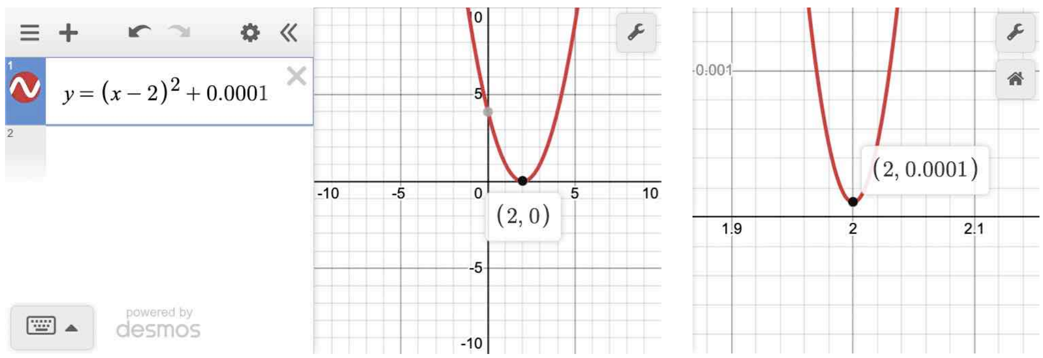
Note 4.3. Besides zooming in and out, the display window can also be set manually via the Graph Settings menu (click on the wrench symbol). The home button resets the window to a window size where the x is approximately between -10 and 10 and with a matching scale for y.

There is also a possibility to rescale each axis individually. To this end hover the pointer over the axis that needs to be rescaled and press and hold the shift key. The axis will appear in blue, and can then be rescaled (click and move the pointer in the wanted direction). Below is the rescaled graph for $y = x^3 - 9x^2 - x$.



Note 4.4. Desmos only approximates its answers, such as intercepts, maxima, and minima. It is *our* task to correctly interpret and confirm any answers inferred from Desmos.

For example, graphing $y = (x-2)^2 + 0.0001$ appears to show a root at $(2, 0)$. Nevertheless, a closer look reveals that this function does *not* have a root at $(2, 0)$.



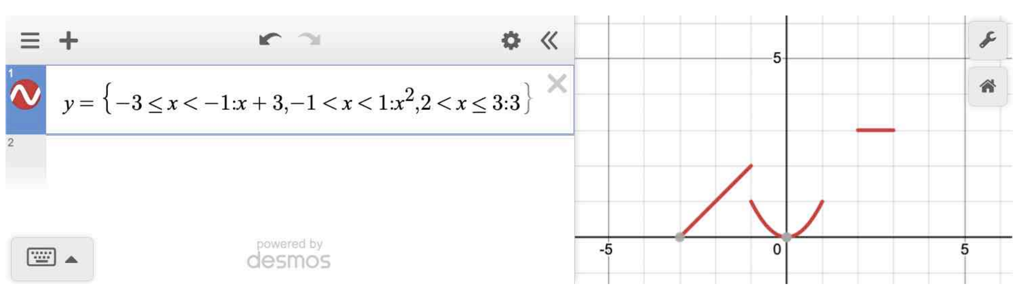
We next show how to graph piecewise defined functions with Desmos.

Example 4.5. Graph the piecewise defined function from Example 3.14 with Desmos.

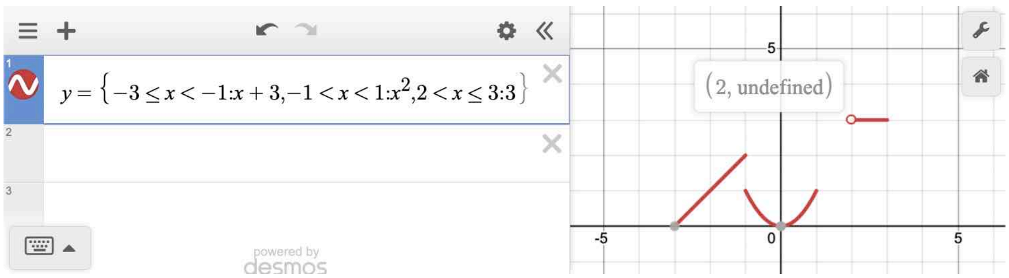
$$y = \begin{cases} x+3 & \text{, for} \quad -3 \le x < -1 \\ x^2 & \text{, for} \quad -1 < x < 1 \\ 3 & \text{, for} \quad 2 < x \le 3 \end{cases}$$

Solution. A piecewise defined function is entered in Desmos with a set bracket $\{\}$, separating each branch with a comma. Each branch is entered as "condition:function value"; for example the top branch in our example is entered as $-3 \le x < -1 : x+3$. Combining the three branches, we obtain:

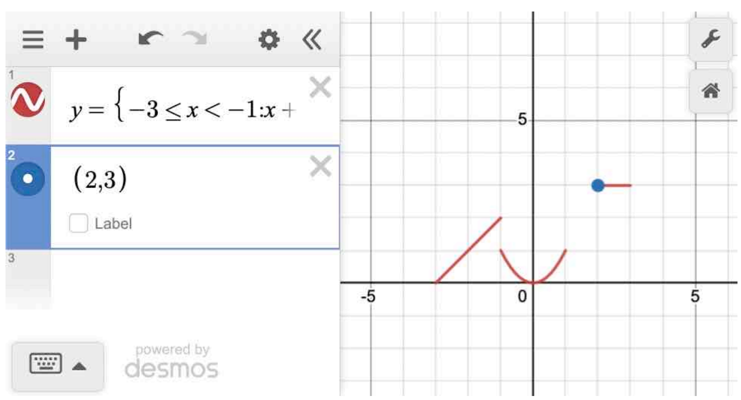
$$y = \{ \quad -3 \le x < -1 : x+3 \quad , \quad -1 < x < 1 : x^2 \quad , \quad 2 < x \le 3 : 3 \quad \}$$



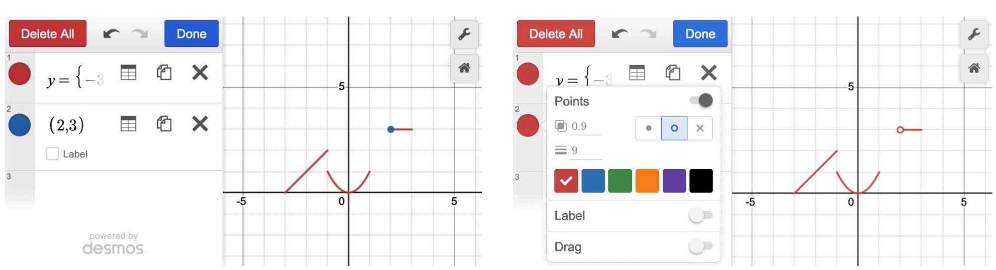
Although there are no open or closed circles at the endpoints of the line segments, Desmos does interpret these endpoints correctly. This can be seen by clicking on the endpoint of a branch.



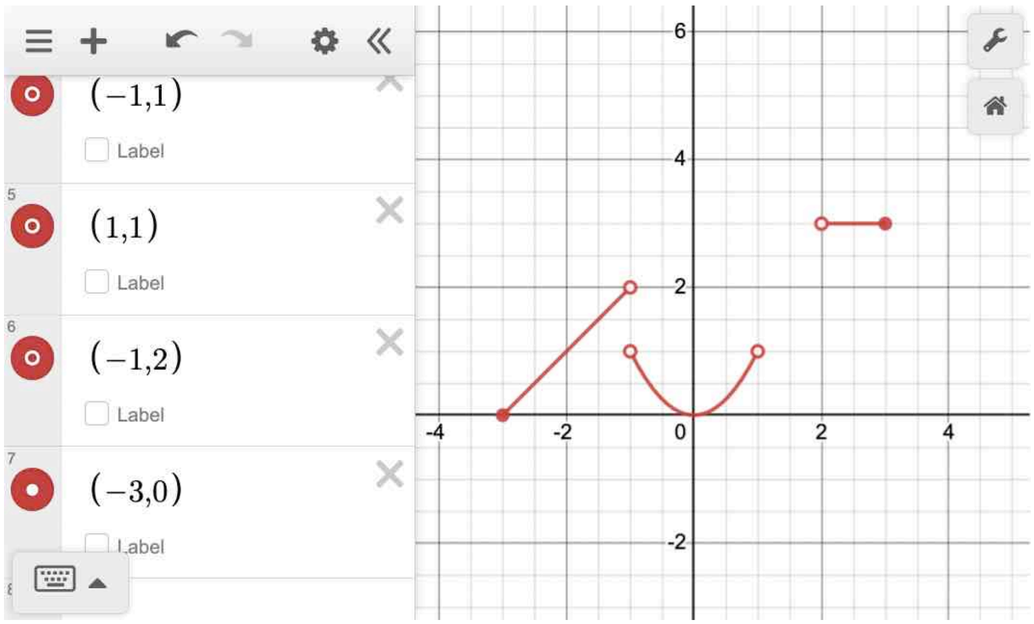
We can add the missing endpoints manually by entering the coordinates for each point.



To adjust the open/closedness of the point, as well as the color, we click the Edit List button ⚙ on top, then click on the big (in this case blue) circle to the left of the entered point. We can then set the style and color of the point.



Performing this for each endpoint, we obtain the following graph.



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Our last example in this section shows how to easily compute function values and how to create tables in Desmos.

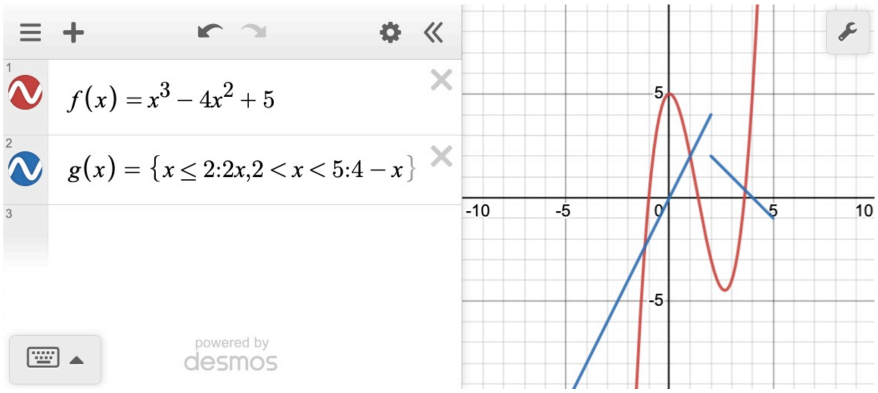
Example 4.6. Consider the functions

$$f(x) = x^3 - 4x^2 + 5$$
$$\text{and} \quad g(x) = \begin{cases} 2x & \text{, for } \quad x \le 2 \\ 4 - x & \text{, for } \quad 2 < x < 5 \end{cases}$$

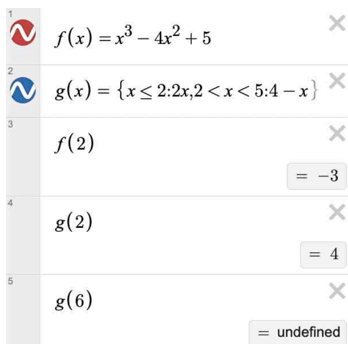
Compute the function values

$$f(2), \qquad f(4), \qquad g(1), \qquad g(2), \qquad g(6).$$

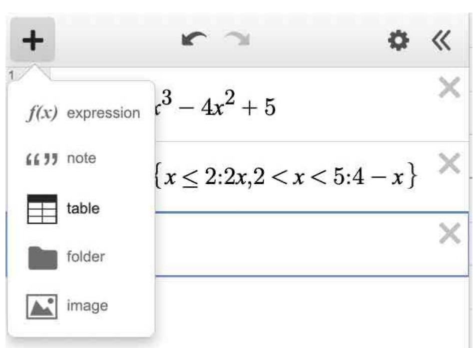
Solution. First, graph both functions f and g.



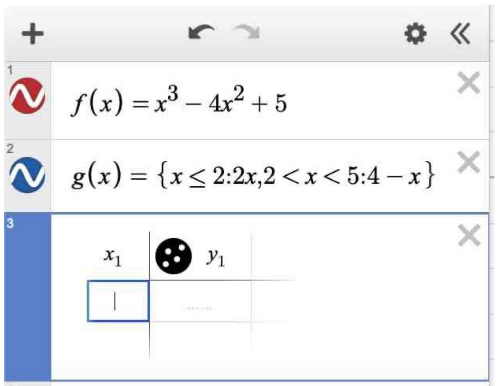
A direct way of computing function values is given by simply entering the wanted expression. Note that undefined function values (such as $g(6)$) are indeed stated as undefined.

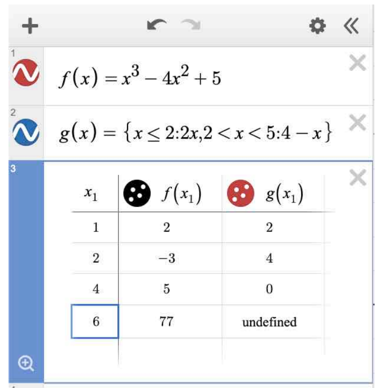


Another way to calculate function values comes from generating a table. Press the Add Item button + on top, and click on “table”.



Modify the table by replacing y_1 with $f(x_1)$. We can also compute multiple function values, such as $f(x_1)$ and $g(x_1)$, by putting $g(x_1)$ next to $f(x_1)$. Below x_1, we enter the desired inputs.





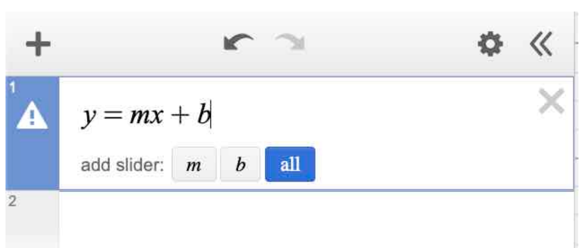
Thus, $f(2) = -3$, $f(4) = 5$, $g(1) = 2$, $g(2) = 4$, and $g(6)$ in undefined. $\square$

4.2 Exploring functions with Desmos

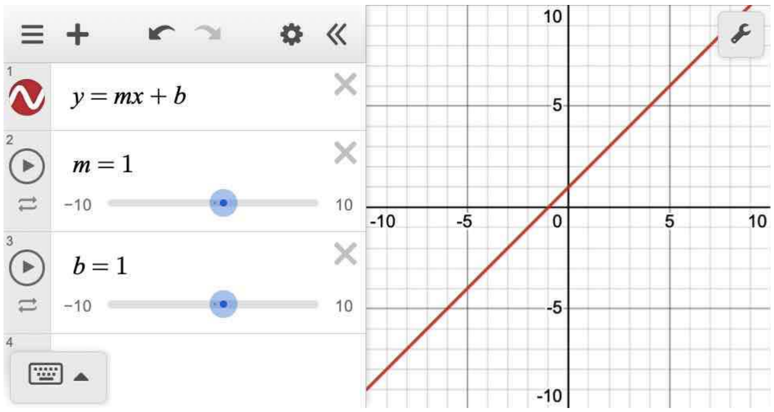
We now explore sliders in Desmos, and we revisit the domain and range of a function, as well as the vertical line test. We also give an example of finding intersection points of two graphs.

Example 4.7. Explore the equation of a line $y = m \cdot x + b$ for various values of m and b using sliders.

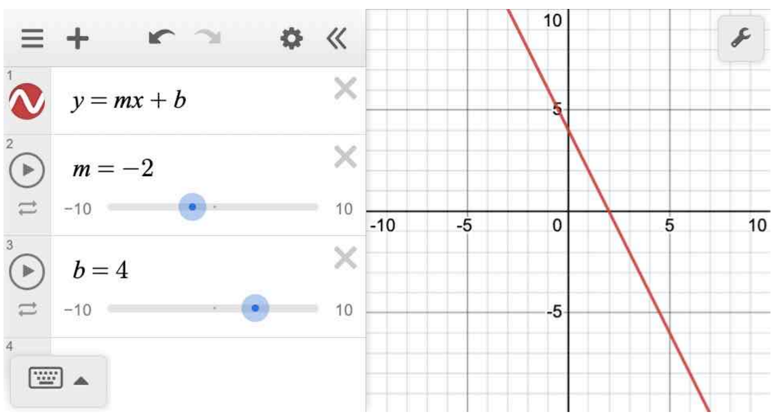
Solution. We enter $y = mx + b$ into Desmos.



We want m and b to be interpreted as constants, but these constants can be adjusted. This is precisely what sliders provide in Desmos. We therefore add the sliders m and b.



Changing the values for m and b, we instantly see the effect on its graph.



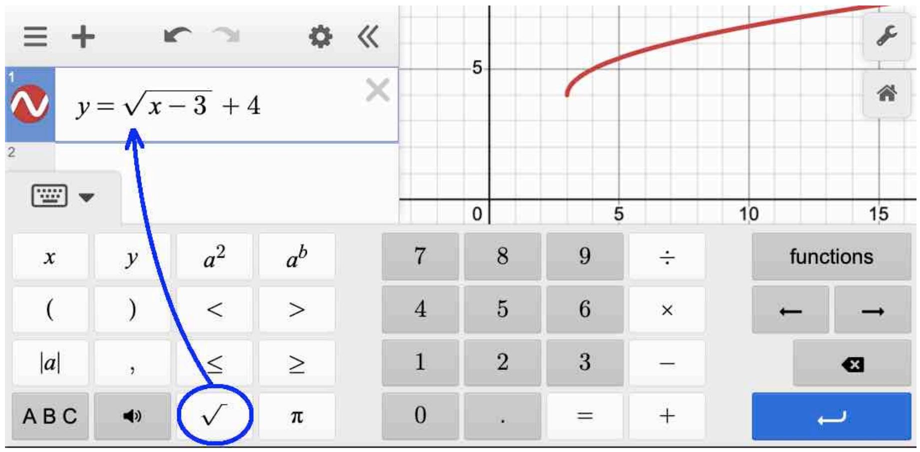
$\square$

Example 4.8. Find the (approximate) domain and range of the following functions.

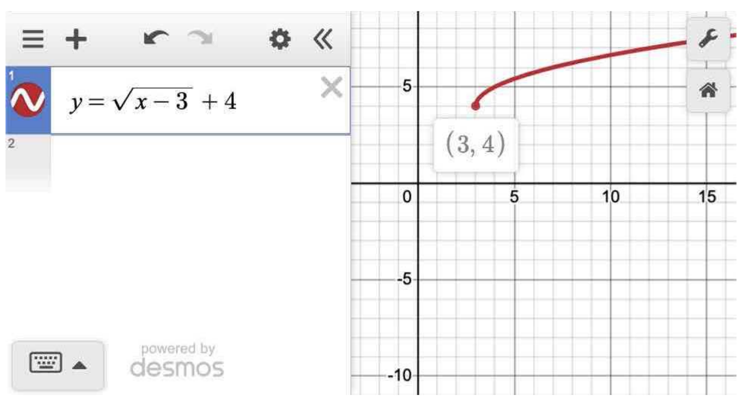
a) $f(x) = \sqrt{x-3} + 4$ b) $f(x) = x^2 + 8x - 7$

Solution.

a) Enter $y = \sqrt{x-3} + 4$ into Desmos. Note, that the square root symbol can be entered by typing the letters $sqrt$, or, alternatively, first show the keyboard by clicking  and then clicking the square-root symbol.

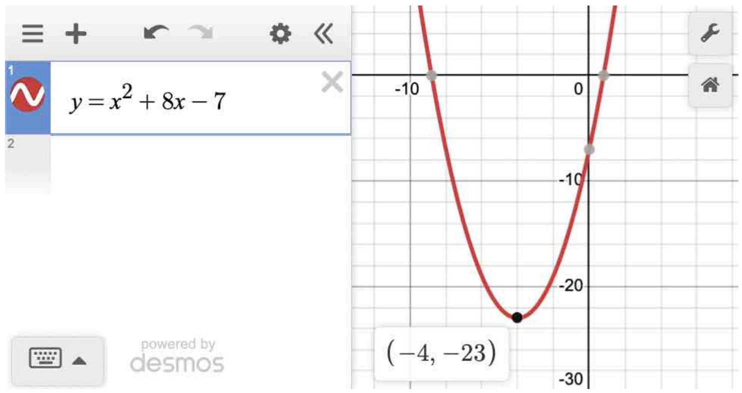


We see that the function starts from some vertex, and then increases as x increases. To find the vertex, click on it.



Thus, the domain and range appear to be $D = [3, \infty)$ and $R = [4, \infty)$.

b) Similarly, we can graph the function $y = x^2 + 8x - 7$ and read off its domain and range. Note, that there is no restriction on the inputs x, so that the domain is all real numbers.



From the graph, the domain and range are therefore $D = \mathbb{R}$ and $R = [-23, \infty)$.

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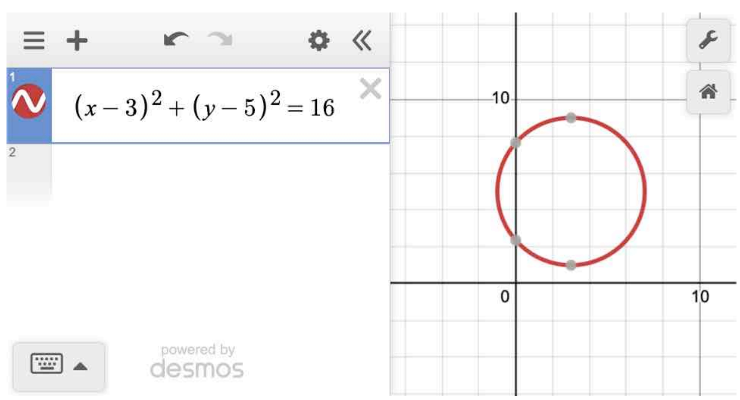
Example 4.9. Graph each of the following equations. Determine whether the graph is the graph of a function or not.

a) $(x-3)^2 + (y-5)^2 = 16$ b) $3x^2 + y^3 + 5xy = 7$

Solution.

a) Graphing $(x-3)^2 + (y-5)^2 = 16$ shows that we obtain a circle.



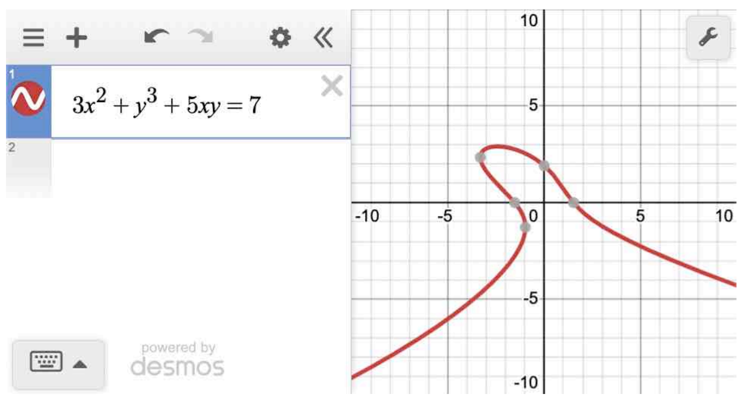
To see if this is the graph of a function, we can use the vertical line test (from Observation 3.10). The y-axis (which is the vertical line at $x = 0$)

intersects the circle in two points. This shows that the circle is not the graph of a function. Indeed, if we solve the equation for y, we get:

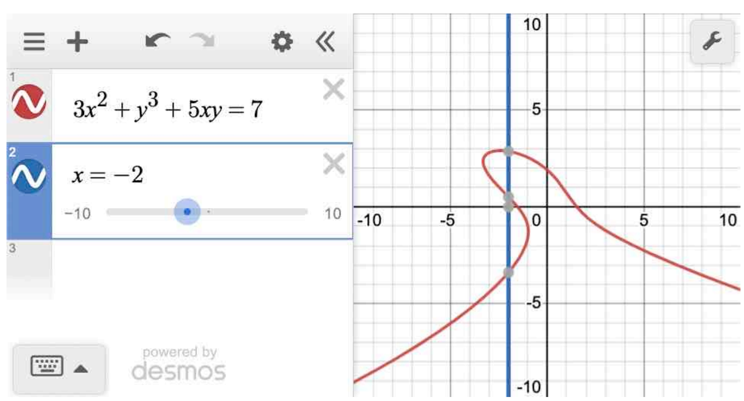
$$\begin{aligned}(x-3)^2+(y-5)^2=16 \quad &\implies \quad (y-5)^2 = 16-(x-3)^2\\ &\implies \quad y-5=\pm\sqrt{16-(x-3)^2}\\ &\implies \quad y=5\pm\sqrt{16-(x-3)^2}\end{aligned}$$

This shows that the circle is made up of two parts, the upper half circle $y=5+\sqrt{16-(x-3)^2}$ and the lower half circle $y=5-\sqrt{16-(x-3)^2}$, each of which *is* the graph of a function.

b) We can easily enter more complicated equations into Desmos.



The vertical line at $x=-2$ intersects the graph at three points, which shows that it is not the graph of a function.



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Note 4.10. We recall that the equation $(x-h)^2+(y-k)^2 = r^2$ always forms a circle in the plane. Indeed, this equation describes a circle with center $C(h,k)$ and radius r. This can easily be explored in Desmos using sliders; see Exercise 4.6 below.

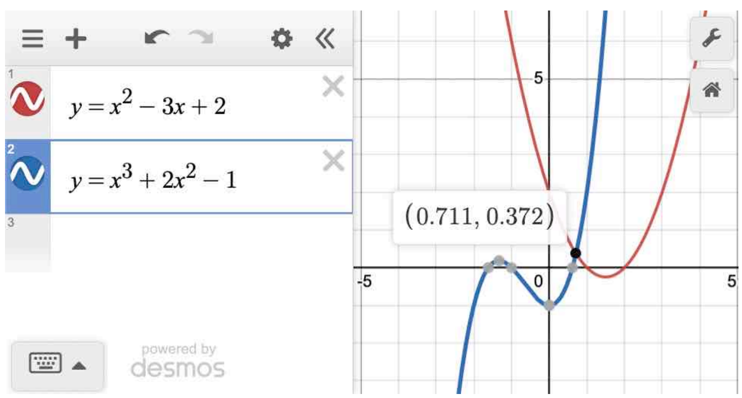
In the last example of this section we solve an equation by determining the intersection of two graphs.

Example 4.11. Solve the equation

$$x^2 - 3x + 2 = x^3 + 2x^2 - 1$$

Approximate your answer to the nearest thousandth.

Solution. We can solve the equation by graphing the left-hand side $y = x^2 - 3x + 2$ and the right-hand side $y = x^3 + 2x^2 - 1$, and by determining those values of x where both sides are equal. This occurs precisely at the intersection of the two graphs. Graphing both functions and clicking on the intersection, we obtain:



The intersection is at $(x,y) \approx (0.711, 0.372)$. Therefore, the two sides of the equation are equal for $x \approx 0.711$ (in which case both the left-hand side and right-hand side are approximately 0.372). Therefore, $x \approx 0.711$ is the approximate solution. $\square$

4.3 Exercises

Exercise 4.1. Graph the function in Desmos.

a) $y = 3x - 5$ b) $y = x^2 - 3x - 2$ c) $y = x^4 - 3x^3 + 2x - 1$
d) $y = \sqrt{x^2 - 4}$ e) $y = \frac{4x+3}{2x+5}$ f) $y = |x+3|$

Exercise 4.2. For each of the functions below, find all roots, all local maxima, all local minima, and the y-intercept.

a) $f(x) = x^3 + 4x^2 - 2x - 9$
b) $f(x) = x^3 - 6x^2 + 7x + 4$
c) $f(x) = -4x^3 + 3x^2 + 7x + 1$
d) $f(x) = 5x^3 + 2x^2$
e) $f(x) = x^4 - x^3 - 4x^2 + 1$
f) $f(x) = -x^4 + 5x^3 - 4x + 3$
g) $f(x) = x^5 + 2x^4 - x^3 - 3x^2 - x$
h) $f(x) = \sqrt{|2^x - 3|} - 2x + 3$

Exercise 4.3. Determine the domain and range using Desmos.

a) $y = |x-2| + 5$
b) $y = -2x + 7$
c) $y = x^2 - 6x + 4$
d) $y = -x^2 - 8x + 3$
e) $y = 3 + \sqrt{x+5}$
f) $y = 6 - x + \sqrt{4-x}$
g) $y = x^4 - 8x^2 + 11$
h) $y = \frac{x-2}{x-3}$

Exercise 4.4. Determine whether the equation describes a function or not.

a) $x^2 + 2y - 3x = 7$
b) $x^2 + 2y^2 - 3x = 7$
c) $y^2 + 8y + 15 = x$
d) $y^3 + x^2 + y + x = 1$
e) $y = \frac{2x-5}{x-3}$
f) $x^2 + \left(y - \sqrt{|x|}\right)^2 = 1$

Exercise 4.5. Solve the equation for y and graph all branches in the same window.

a) $x^2 + y^2 = 4$
b) $(x+5)^2 + y^2 = 15$
c) $(x-1)^2 + (y-2)^2 = 9$
d) $y^2 = x^2 + 3$
e) $y^2 + x^2 - 8x - 14 = 0$
f) $y^2 = -x^2 + 77$

Exercise 4.6. Set up the general equation of a circle in Desmos, where the center and the radius can be changed using sliders. If a circle of radius 3 with center at the origin $(0,0)$ is shifted 4 units to the right and shifted 2 units down, then what is its equation?

Exercise 4.7. Find all solutions of the equation. Round your answer to the nearest thousandth.

a) $x^3 + 3 = x^5 + 7$
b) $4x^3 + 6x^2 - 3x - 2 = 0$
c) $\frac{2x}{x-3} = \frac{x^2+2}{x+1}$
d) $5^{3x+1} = x^5 + 6$
e) $x^3 + x^2 = x^4 - x^2 + x$
f) $3x^2 = x^3 - x^2 + 3x$