Algebra, Functions, Calculus, and Pedagogical Content Knowledge

CST Workshop Summer 2016

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City University of New York

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Exercise. Which of these demonstrate the relationship between the sets of prime numbers, real numbers, natural numbers, complex numbers, rational numbers, and integers? $\mathbb{P}$ = Prime, $\mathbb{R}$ = Real, $\mathbb{N}$ = Natural, $\mathbb{C}$ = Complex, $\mathbb{Q}$ = Rational, $\mathbb{Z}$ = Integer.

a. $\mathbb{P} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{Z} \subseteq \mathbb{C} \subseteq \mathbb{N}$

b. $\mathbb{P} \subseteq \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

c. $\mathbb{C} \subseteq \mathbb{R} \subseteq \mathbb{Q} \subseteq \mathbb{Z} \subseteq \mathbb{N} \subseteq \mathbb{P}$

d. None of these

Exercise. Which of the following products would result in a rational number?

a. -4×6

b. $-\sqrt{5} \times 1$

c. $e \times 2^3$

Exercise. If the square of twice the sum x and three is equal to the product of twenty-four and x, which of these is a possible value of x?

a. $6 + 3\sqrt{2}$
b. $\frac{2}{3}$
c. $-3i$
d. -3

Exercise. Solve $7x^2 + 6x = -2$.

a. $x = \frac{-3 \pm \sqrt{23}}{7}$

b. $x = \pm i\sqrt{5}$

c. $x = \pm \frac{2i\sqrt{2}}{7}$

d. $x = \frac{-3 \pm i\sqrt{5}}{7}$

Exercise. Solve $x^4 + 64 = 20x^2$.

a. $x = \{2, 4\}$

b. $x = \{-2, 2, -4, 4\}$

c. $x = \{2i, 4i\}$

d. $x = \{-2i, 2i, -4i, 4i\}$

Exercise. Simplify $\frac{2+3i}{4-2i}$.

a. $\frac{1}{10} + \frac{4}{5}i$

b. $\frac{1}{10}$

c. $\frac{7}{6} + \frac{2}{3}i$

d. $\frac{1}{10} + \frac{3}{10}i$

Exercise. Simplify $|(2-3i)^2-(1-4i)|$.

a. $\sqrt{61}$

b. $-6-8i$

c. $6+8i$

d. 10

Exercise. Simplify

$$\sqrt{\frac{-28x^6}{27y^5}}$$

a. $\frac{2x^3 i\sqrt{21y}}{9y^3}$

b. $\frac{2x^3 i\sqrt{21y}}{27y^4}$

c. $\frac{-2x^3\sqrt{21y}}{9y^3}$

d. $\frac{12x^3 yi\sqrt{7}}{27y^2}$

Exercise. Simplify

$$\frac{(x^2y)(2xy^{-2})^3}{16x^5y^2} + \frac{3}{xy}$$

a. $\frac{3x+24y^6}{8xy^7}$

b. $\frac{x+6y^6}{2xy^7}$

c. $\frac{x+24y^5}{8xy^6}$

d. $\frac{x+6y^5}{2xy^6}$

Exercise. Solve

$$4|2x+6|-5<27.$$

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3. Let A be a 4×3 matrix and AB a 4×6, then what if the size of B?

Exercise. For vector $\mathbf{v} = (4, 3)$ and vector $\mathbf{w} = (-3, 4)$, find $2(\mathbf{v} + \mathbf{w})$.

a. $(2, 14)$

b. $(14, -2)$

c. $(1, 7)$

d. $(7, -1)$

Exercise. Simplify

$$\begin{pmatrix} 2 & 0 & -5 \end{pmatrix}\left(\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ -5 \end{pmatrix}\right)$$

Exercise. Solve the equation $x^2 - 36 - 10x = 0$ by completing the square.

Exercise. Derive the quadratic formula for $ax^2 + bx + c = 0$ by completing the square.

Exercise. Solve the following quadratic equations.

a. $x^2 + 2x - 8 = 0$

b. $3x^2 = 10x - 5$

c. $5 = -x^2 - 4x$

Exercise. The roots of the equation $5x^2 - 2x + 1 = 0$ are

a. Real, rational, and unequal.
b. Real, rational, and equal.
c. Real, irrational, and unequal.
d. Imaginary.

Systems of Linear Equations

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Exercise. Solve the following system of equations algebraically.

$$\begin{aligned} 3x - y &= 2, \\ x + y &= -6. \end{aligned}$$

Systems of Linear Equations

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Exercise. Solve the following system of equations.

$$\begin{aligned} 2x - 4y &= 4, \\ 3x + 2y &= 14. \end{aligned}$$

Nonlinear System of Equations

NONLINEAR SYSTEM OF EQUATIONS

Exercise. Solve the following system of equations algebraically.

$$
\begin{aligned}
x^2 + y^2 &= 25 \\
2y - x &= 5
\end{aligned}
$$

Nonlinear System of Equations

Exercise. Solve the following system of equations algebraically.

$$\begin{aligned} \frac{x+y}{x+3} &= x-2 \\ y &= x+6 \end{aligned}$$

RATIONAL EQUATIONS

Exercise. What is the solution set of the equation

$$\frac{x+8}{x^2-4} - \frac{5}{x^2-2x} = \frac{2}{x}\ ?$$

Rational Equations

Exercise. Solve for x and express your answer in simplest radical form:

$$\frac{4}{x} - \frac{3}{x+1} = 7.$$

Absolute Value Equations

Absolute Value Equations

Recall: $$|a| = \begin{cases} a, & \text{if } a \geqslant 0, \\ -a, & \text{if } a < 0. \end{cases}$$

Absolute Value Equations

Exercise. What is the solution set of the equation $|x+2| = 7$?

Absolute Value Equations

Exercise. What is the solution set of the equation $|2x-3|-4=3$?

ABSOLUTE VALUE EQUATIONS

Exercise. What is the solution set of the equation $|x^2 - 2x| = 3x - 6$?

Square Root Equations

Exercise. Solve $2 - \sqrt{x} = \sqrt{x - 20}$

a. $x = 6$

b. $x = 36$

c. $x = 144$

d. No solution

Geometric Sequences

GEOMETRIC SEQUENCES

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$$a + ar + ar^2 + \cdots + ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right)$$

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GEOMETRIC AND ARITHMETIC SEQUENCES

Geometric and Arithmetic Sequences

Exercise. An arithmetic sequence has a first term of 10 and a sixth term of 40. What is the 20th term of this sequence?

a. 105

b. 124

c. 110

d. 130

SEQUENCES

SEQUENCES

Exercise. The first four terms of the sequence defined by $a_1 = \frac{1}{2}$ and $a_{n+1} = 1 - a_n$ are

a. $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

b. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

c. $\frac{1}{2}, 1, 1\frac{1}{2}, 2$

d. $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$

Sequences

SEQUENCES

Exercise. The common ration of the sequence $-\frac{1}{2}, \frac{3}{4}, -\frac{9}{8}$ is

a. $-\frac{3}{2}$

b. $-\frac{1}{2}$

c. $-\frac{2}{3}$

d. $-\frac{1}{4}$

Exponential and Logarithmic Functions

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An exponential function is a function of the form

$$y = a^x,$$

where a is positive and not equal to 1.

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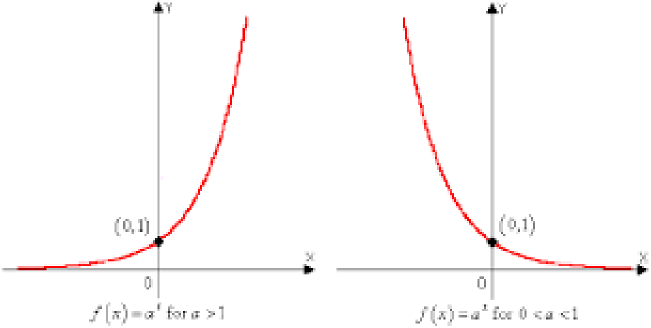

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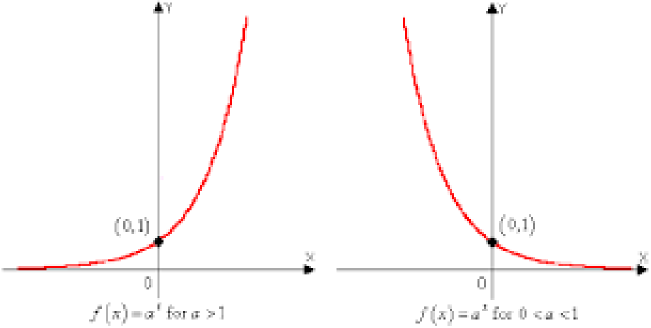

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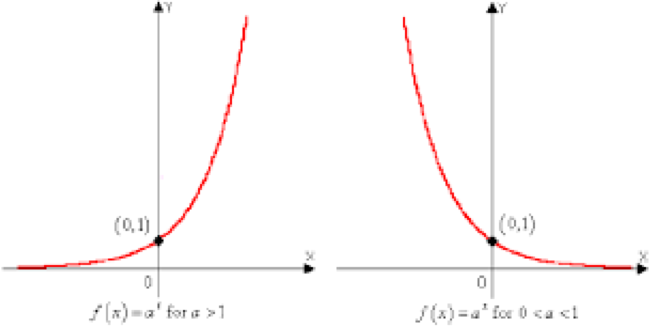

What is the domain of $y = a^x$? What is the range of $y = a^x$?

Exponential and Logarithmic Functions

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The logarithmic function is the function $y = \log_b x$, where b is any number such that $b > 0$, $b \neq 1$, and $x > 0$.

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Properties:

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Properties:

1. $\log_b 1 = 0$.
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EXPONENTIAL AND LOGARITHMIC FUNCTIONS

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Exercise. Solve for x: $\log_4(x^2 + 3x) - \log_4(x + 5) = 1$.

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Exercise. If $\log_b x = y$, the what does x equal?

Exponential and Logarithmic Functions

Exercise. Growth of a certain strain of bacteria is modeled by the equation $G = Ae^{0.58t}$, where:

$G =$ final number of bacteria,
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EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Exercise. Growth of a certain strain of bacteria is modeled by the equation $G = Ae^{0.58t}$, where:

G = final number of bacteria,
A = initial number of bacteria,
t =time in hours.

In how many hours will 4 bacteria increase to approximately 2500 bacteria? Round your answer to the nearest hundredth of an hour.

TRANSFORMATIONS OF FUNCTIONS

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c. $f(x) + a$

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e. $f(-x)$

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e. $f(-x)$ Reflection over the y-axis.

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g. $af(x)$

Transformations of Functions

Using GeoGebra, we will explore the effects of the following transformations ($a > 0$):

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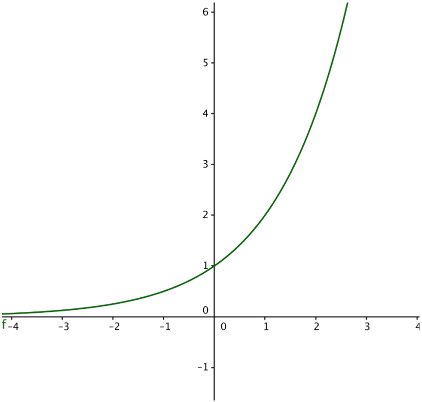
d. $f(x)-a$ Vertical translation by a units down.
e. $f(-x)$ Reflection over the y-axis.
f. $-f(x)$ Reflection over the x-axis.
g. $af(x)$ Dilation by a factor of a.

Exercise. Write the equation for the graph of function $g(x)$, obtained by shifting the graph of $f(x) = x^2$ three units left, stretching the graph vertically by a factor of two, reflecting that result over the x-axis, and then translating the graph up four units.

Exercise. Write the equation for the graph of function $g(x)$, obtained by shifting the graph of $f(x) = x^2$ three units left, stretching the graph vertically by a factor of two, reflecting that result over the x-axis, and then translating the graph up four units.

$$g(x) = -2(x+3)^2 + 4$$

Exercise. Which of these could be the equation of the function graphed below?



a. $f(x) = x^2$
b. $f(x) = \sqrt{x}$
c. $f(x) = 2x$
d. $f(x) = \log_2 x$

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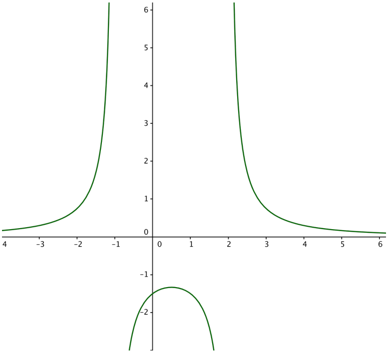


a. $f(x) = \frac{3}{x^2-x-2}$

b. $f(x) = \frac{3x+3}{x^2-x-2}$

c. $f(x) = \frac{1}{x+1} + \frac{1}{x-2}$

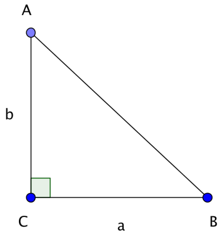
d. None of these

Trigonometric Identities

Exercise. Prove that the equation shown below is an identity for all values for which the functions are defined:

$$\csc\theta \cdot \sin^2\theta \cdot \cot\theta = \cos\theta$$

Exercise. For the right triangle below, which of the following is a true statement of equality?



a. $\tan B = \frac{a}{b}$

b. $\cos B = \frac{a\sqrt{a^2+b^2}}{a^2+b^2}$

c. $\sec B = \frac{\sqrt{a^2+b^2}}{b}$

d. $\csc B = \frac{a^2+b^2}{b}$

Exercise. What is the exact value of $\tan(-\frac{2\pi}{3})$?

a. $\sqrt{3}$
b. -$\sqrt{3}$
c. $\frac{\sqrt{3}}{3}$
d. 1

Exercise. If $\sin\theta = \frac{1}{2}$ when $\frac{\pi}{2} < \theta < \pi$, what is the value of θ?

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. $\frac{2\pi}{3}$
d. $\frac{5\pi}{6}$

Exercise. Which of the following expressions is equal to $\cos\theta\cot\theta$?

a. $\sin\theta$
b. $\sec\theta\tan\theta$
c. $\csc\theta - \sin\theta$
d. $\sec\theta - \sin\theta$

LIMITS

Exercise. Evaluate the limit

$$\lim_{x \to -3} \frac{x^3 + 3x^2 - x - 3}{x^2 - 9}$$

a. 0
b. $\frac{1}{3}$
c. $\frac{-4}{3}$
d. ∞

LIMITS

Exercise. Evaluate the limit

$$\lim_{x\to\infty} \frac{x^2+2x-3}{2x^2+1}$$

a. 0
b. $\frac{1}{2}$
c. -3
d. ∞

LIMITS

Exercise. Evaluate the limit

$$\lim_{x \to 3^+} \frac{|x-3|}{3-x}$$

a. 0
b. -1
c. 1
d. ∞

LIMITS

Exercise. If $f(x) = 2x^3 - 3x^2 + 4$, what is

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

a. -4

b. 4

c. 8

d. 12

DERIVATIVES

Derivatives

- Product Rule: $(fg)' = f'g + g'f$

Derivatives

- Product Rule: $(fg)' = f'g + g'f$
- Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

DERIVATIVES

- Product Rule: $(fg)' = f'g + g'f$
- Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
- Chain Rule: $[f(g(x))]' = f'(g(x))g'(x)$

DERIVATIVES

Exercise. Find the derivative of $f(x) = e^{3x^2-1}$

a. $6xe^{6x}$
b. e^{3x^2-1}
c. $(3x^2-1)e^{3x^2-2}$
d. $6xe^{3x^2-1}$

DERIVATIVES

Exercise. Find the derivative of $f(x) = \ln(2x+1)$

a. $\frac{1}{2x+1}$
b. $2e^{2x+1}$
c. $\frac{2}{2x+1}$
d. $\frac{1}{2}$

Derivatives

Exercise. If $f(x) = \frac{1}{4}x^2 - 3$, find the slope of the line tangent to the graph of $f(x)$ at $x = 2$.

a. -2
b. 0
c. 1
d. 4

Derivatives

Exercise. If $f(x) = 4x^3 - x^2 - 4x + 2$, which of the following statements is/are true of its graph?

I. The point $(-\frac{1}{2}, 3\frac{1}{4})$ is a relative maximum.
II. The graph of f is concave upward on the interval $(-\infty, \frac{1}{2})$.

a. I
b. II
c. I and II
d. Neither I nor II

INTEGRALS

Exercise. Calculate $\int 3x^2 + 2x - 1\, dx$

a. $x^3 + x^2 - x + c$
b. $6x^2 + 2$
c. $\frac{3}{2}x^3 + 2x^2 - x + c$
d. $6x^2 + 2 + c$

Integrals

Exercise. Calculate $\int 3x^2 e^{x^3}\, dx$

a. $x^3 e^{x^3} + c$

b. $e^{x^3} + c$

c. $x^3 e^{\frac{x^4}{4}} + c$

d. $\ln x^3 + c$

PCK SAMPLE QUESTION

Use the information below to complete the task that follows.

As a mathematics teacher, you are preparing to teach a lesson to address an aspect of the following standard from the New York State P–12 Common Core Learning Standards:

> **Interpreting Functions (F–IF)**
> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, and using technology for more complicated cases.
>
> a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Prepare a response of approximately 400–600 words in which you:

- state the student learning objectives for this lesson;
- describe the conceptual understanding, skills, and prerequisite knowledge that students need in order to understand the content described by this learning standard;
- describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;
- describe a method for helping students build a viable argument related to the learning standard; and
- describe a method for assessing students' progress toward the goal of understanding the content described by the learning standard.

PCK Sample Question

state the student learning objectives for this lesson;

PCK Sample Question

state the student learning objectives for this lesson;

Students will plot graphs of linear and quadratic functions by hand and by using a graphing calculator.

Students will compare and analyze graphs to identify key features (degree of function, intercepts, maxima, minima, increasing, decreasing, etc.).

PCK Sample Question

describe the conceptual understanding, skills, and prerequisite knowledge that students need in order to understand the content described by this learning standard;

PCK SAMPLE QUESTION

describe the conceptual understanding, skills, and prerequisite knowledge that students need in order to understand the content described by this learning standard;

To understand the new content that will be addressed in this lesson, students will need prerequisite knowledge that a graph is the set of solutions for a given equation, as well as the general shape of a linear and quadratic graph. They must know how to determine the degree of a polynomial. Students will require the skills to plot points on the coordinate plane and connect the points to form a graph and skills in using a graphing calculator to graph an equation in $y =$ format. Conceptually, students will need to understand the relationship between the function and the graph.

PCK Sample Question

describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;

PCK Sample Question

describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;

An instructional strategy to achieve the learning objectives would be guided instruction. I will begin by showing the graphs, $y = 2x + 1$ and $y = x^2 + 4x + 4$, and lead a discussion asking students what they see as similarities/differences in the two graphs: one linear, one quadratic. Terms such as x-intercept, y-intercept, vertex, etc. will be defined and students would identify these on the sample graphs. Throughout discussion, students will view additional graphs of linear and quadratic functions, identifying the type of function each graph displays and locating intercepts for each and the vertex for the parabolas. I would check for understanding as students explained how they determined which type of function each graph was, and how they located the key features. Students would demonstrate knowledge of similarities/differences between all the linear graphs (some increasing, some decreasing, number of x- and y-intercepts) and between all the quadratic graphs (some have a maximum, some a minimum for a vertex, number of x- and y-intercepts the graph has).

PCK Sample Question

describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;

PCK Sample Question

describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;

Following discussion, each student would receive a worksheet with a combination of 5 linear and quadratic equations. First, they will insert each function into the graphing calculator and sketch the appropriate graph next to the given equations. Students will identify whether the graph is linear or quadratic, based on shape. Students would record their observations about the degree of the equations (linear are first degree; quadratic are second degree). The next task on the worksheet would include a different set of equations, both linear and quadratic. Students would predict the type of graph each equation will produce, complete a data table by evaluating the equation for given x values, and plot the graph. Students will use the graph to identify its intercepts and maximum or minimum point.

PCK Sample Question

describe a method for helping students build a viable argument related to the learning standard;

PCK Sample Question

describe a method for helping students build a viable argument related to the learning standard;

To help students build a viable argument, students would be assigned a partner to compare work, confirm understanding and discuss questions like: What features in an equation reveal whether a line or a parabola will be formed? What are the similarities and differences between linear and quadratic graphs?

PCK Sample Question

describe a method for assessing students? progress toward the goal of understanding the content described by the learning standard.

PCK SAMPLE QUESTION

describe a method for assessing students? progress toward the goal of understanding the content described by the learning standard.

Assessment would be ongoing throughout the lesson. I would check for understanding at the beginning, during discussion, to ensure students are ready for the independent work. I would also circulate among students during the independent work and paired discussion to assist students who were struggling or offer a challenging extension for students who finished before others and showed mastery of the learning objectives. At the close of class, I would engage students in a whole group discussion to summarize what was learned about key features of linear and quadratic graphs and different methods of graphing them. The worksheet would be turned in at the end of class so I could evaluate student work and use it to inform future instruction.