Introduction
CST Workshop Summer 2016

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City University of New York

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- Note: Don't forget to bring valid ID on the day of the test (E.g., driver's license, passport)

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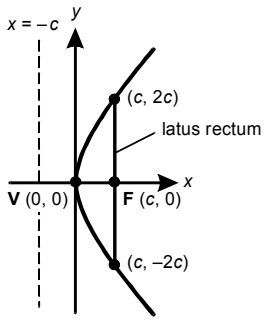
- We recommend that you register at least 30 days in advance. Because test appointments are accepted on a first-come, first-served basis and seating is limited, you may not be able to schedule a test appointment less than a week in advance of your desired test date, or within a week of a testing window.

Definitions and Formulas for Mathematics

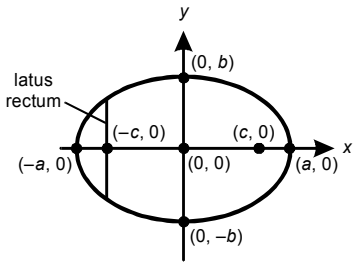
GEOMETRY

Parabola



$(y - k)^2 = 4c(x - h)$

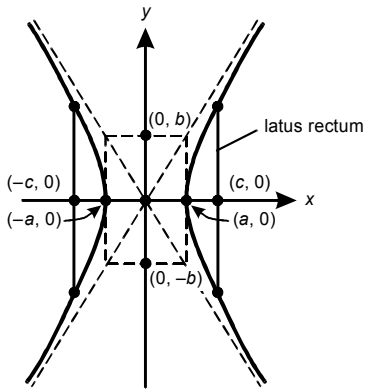
Ellipse



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where $c^2 = a^2 - b^2$

Hyperbola



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

where $b^2 = c^2 - a^2$

Eccentricity of a Conic

$$e = \frac{c}{a}$$

TRIGONOMETRY

$$\sin(\theta_1 \pm \theta_2) = \sin\theta_1 \cos\theta_2 \pm \cos\theta_1 \sin\theta_2$$

$$\cos(\theta_1 \pm \theta_2) = \cos\theta_1 \cos\theta_2 \mp \sin\theta_1 \sin\theta_2$$

$$\tan(\theta_1 \pm \theta_2) = \frac{\tan\theta_1 \pm \tan\theta_2}{1 \mp \tan\theta_1 \tan\theta_2}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

NOTES FOR MATHEMATICS TEST

In this examination, assume all functions are real valued functions unless otherwise noted.

In this examination, diagrams may not be drawn to scale.

Approved Calculators

- Texas Instruments TI-73
- Texas Instruments TI-83
- Texas Instruments TI-83 Plus
- Texas Instruments TI-83 Plus Silver Edition
- Texas Instruments TI-84
- Texas Instruments TI-84 Plus
- Texas Instruments TI-84 Plus C Silver Edition
- Texas Instruments TI-84 Plus Silver Edition
- Texas Instruments TI-Nspire Handheld with the TI-84 Plus Keypad (Examinees using the TI-Nspire Handheld may not remove the TI-84 Plus Keypad while testing. Violation of the rule, or any other test site rule, may result in the voiding of your scores.)

Competency Areas addressed on CST:

Competency Areas addressed on CST:

- Number and Quantity 8%
- Algebra 20%
- Functions 17%
- Calculus 10%
- Geometry and Measurement 15%
- Statistics and Probability 10%
- Pedagogical Content Knowledge 20% (constructed-response)

A few references that may be useful for review:

- Regents Exam Prep Center http://www.regentsprep.org
- NYSTCE Mathematics (04) Study Guide: Test Prep and Practice Questions, Trivium Test Prep $39.99.
- NYSTCE Mathematics (004) Test Secrets Study Guide: NYSTCE Exam Review for the New York State Teacher Certification Examinations, NYSTCE Exam Secrets Test Prep Team $62.99.

Number and Quantity

- applies and extends understanding of arithmetic to the rational numbers
- applies properties of rational numbers to solve real-world and mathematical problems involving the four operations with rational numbers
- applies and extends understanding of integer exponents to include rational exponents and rewrites expressions involving radicals and rational numbers
- reasons quantitatively and uses appropriate units to solve problems
- demonstrates understanding of the properties of real numbers and applies real numbers to model and solve multistep problems

NUMBER AND QUANTITY

- performs arithmetic operations with complex numbers
- represents complex numbers and their operations in the complex plane, using both rectangular and polar forms
- uses complex numbers to factor and solve quadratic equations and applies the fundamental theorem of algebra
- models and solves problems with vector quantities, including problems involving velocity and other quantities represented by vectors
- performs arithmetic operations (e.g., addition, subtraction, scalar multiplication) on vectors and represents vectors, their magnitudes, and vector operations symbolically and graphically
- demonstrates understanding of the properties of matrices, performs operations on matrices, and uses matrices in applications
- demonstrates knowledge of abstract algebra (e.g., groups, rings, fields, vector spaces)

NUMBER AND QUANTITY SAMPLE QUESTION

1. Use the information below to answer the question that follows.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

Matrix B is composed of the elements from matrix A, as shown above. Which statement describes the relationship between the determinants of the two matrices?

a. $\det(B) = \det(A)$

b. $\det(B) = -\det(A)$

c. $\det(B) = \frac{1}{\det(A)}$

d. $\det(B) = -\frac{1}{\det(A)}$

Algebra

- uses properties of operations to generate equivalent expressions and solves real- life and mathematical problems using numerical and algebraic expressions and equations
- analyzes rates and proportional relationships and uses them to solve real-world and mathematical problems
- analyzes connections between proportional relationships, lines, and linear equations
- interprets the structure of expressions and rewrites expressions in equivalent forms (e.g., factoring, completing the square in a quadratic expression, transforming exponential expressions and equations, finding the sum of a finite geometric series)
- performs arithmetic operations on polynomials, simplifies polynomial expressions using identities, and expands binomials
- demonstrates understanding of the relationship between zeros and factors of polynomials and extends polynomial identities to the complex numbers
- rewrites and manipulates rational expressions

Algebra

- creates equations and inequalities in one, two, or more variables to describe numbers or relationships (e.g., linear, quadratic, exponential), including situations involving constraints, and interprets the viability of options in modeling contexts
- understands solving equations and inequalities as a process of reasoning and explains the reasoning, including situations when extraneous solutions may arise
- solves linear equations and inequalities and quadratic equations in one variable
- solves systems of linear and quadratic equations using a variety of methods (e.g., algebraic, graphic, matrix)
- represents and solves linear and nonlinear equations and inequalities graphically

Algebra Sample Question

Algebra Sample Question

2. Which equation could be a step in finding the points of intersection of the graphs of the curves given by $y = 2x^2 - 2x + 8$ and $y = 4x^2 - 6x + 3$ when using the method of completing the square?

a. $(x-2)^2 = \frac{5}{2}$
b. $(x-1)^2 = \frac{6}{2}$
c. $(x-1)^2 = \frac{7}{2}$
d. $(x-2)^2 = \frac{9}{2}$

FUNCTIONS

- demonstrates understanding of the concept of a function and the use of function notation, including sequences and recursive functions
- interprets functions that arise in applications in terms of context and interprets key features (e.g., domain, intercepts, rate of change, end behavior, periodicity) of functional relationships presented in written descriptions, symbolic expressions, tables, or graphs
- uses different representations to analyze functions (e.g., linear, quadratic, radical, rational, piecewise, absolute value, exponential, logarithmic)
- builds functions that model relationships between two quantities using a variety of methods (e.g., explicit expressions, recursive processes, arithmetic combination of functions, composition of functions)
- analyzes arithmetic and geometric sequences both recursively and with an explicit formula, translates between the two forms, and uses them to model situations

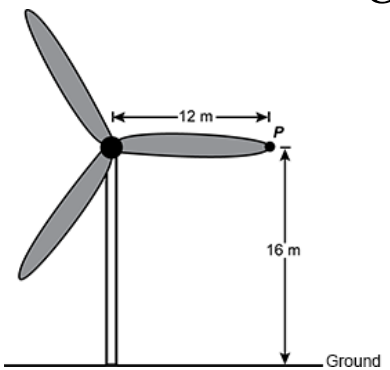
FUNCTIONS

- builds new functions from existing functions and analyzes their graphs (e.g., analyzes the effect of replacing $f(x)$ with $f(x)+k, kf(x), f(kx)f(x+k)$), finds and analyzes inverse functions, and identifies even and odd functions
- compares and contrasts linear, quadratic, and exponential functions, and uses them to model and solve problems
- solves problems involving logarithmic and exponential functions
- analyzes trigonometric functions using the unit circle
- models periodic phenomena with trigonometric functions
- proves and applies trigonometric identities
- solves trigonometric equations

Function Sample Question

FUNCTION SAMPLE QUESTION

3. Use the diagram below to answer the question that follows.



The diagram above shows a wind turbine on a vertical shaft. The height of the shaft is 16 m. The length of a wind turbine blade is 12 m. Point P is located at the tip of one of the wind turbine blades. The wind turbine rotates in the counterclockwise direction at a constant rate of 1 revolution per second. At $t = 0$, point P is located at a height of 16 m above ground. Which function models the height of point P above ground as a function of t?

a. $f(t) = 12\cos(2\pi t)$

b. $f(t) = 16\cos(\pi t)$

c. $f(t) = 12\sin(2\pi t) + 16$

d. $f(t) = 16\sin(\pi t) + 4$

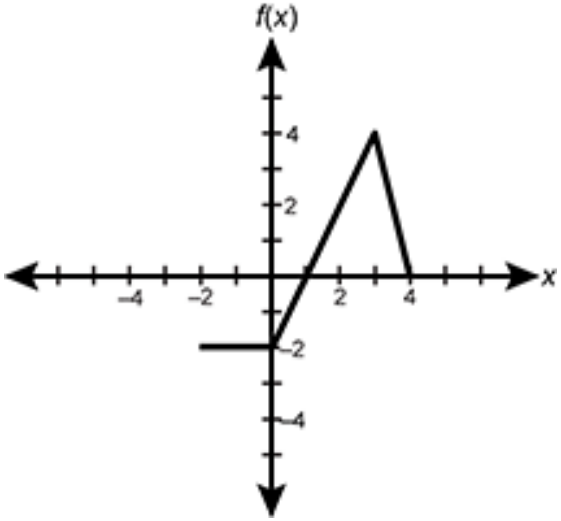
CALCULUS

- analyzes the concept of limits and applies it to interpret the properties of functions (e.g., continuity, asymptotes)
- interprets derivatives and definite integrals as limits (e.g., difference quotients, slope, Riemann sums, area)
- applies the fundamental theorem of calculus
- applies techniques of differentiation and integration (e.g., product rule, chain rule, u-substitution)
- applies properties of derivatives to analyze the graphs of functions
- demonstrates knowledge of power series
- uses derivatives to model and solve mathematical and real-world problems (e.g., rates of change, related rates, optimization)
- uses integration to model and solve mathematical and real-world problems (e.g., work, applications of antiderivatives)
- models and solves problems involving first order differential equations (e.g., separation of variables, initial value problems)

Calculus Sample Question

CALCULUS SAMPLE QUESTION

4. Use the diagram below to answer the question that follows.



The graph of a function $f(x)$ is shown above. What is the value of $\int_{-2}^{4} f(x)dx$.

a. 1

b. 5

c. 6

d. 11

Geometry and Measurement

- understands the Pythagorean theorem and its converse (including proofs) and applies the theorem to solve problems in two and three dimensions and in the coordinate plane
- analyzes and applies properties of rotations, reflections, and translations in the plane and demonstrates understanding of congruence in terms of rigid motions
- proves and applies theorems about lines and angles, triangles, and parallelograms
- proves and analyzes geometric constructions
- demonstrates understanding of similarity in terms of similarity transformations and proves theorems involving similarity
- applies right triangle trigonometry to solve problems, and applies trigonometry to general triangles (e.g., law of sines, law of cosines)
- applies theorems about circles and solves measurement problems involving circles (e.g., arc lengths, areas of sectors)

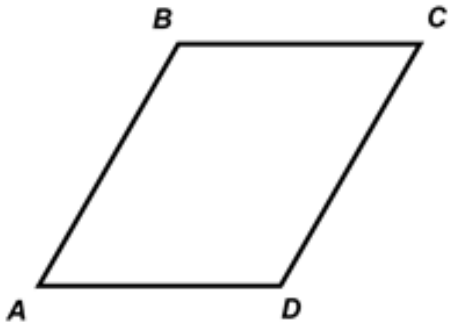
Geometry and Measurement

- translates between geometric descriptions and equations for conic sections (e.g., circles, parabolas, ellipses, hyperbolas)
- uses coordinates to prove simple geometric theorems algebraically (e.g., slope criteria for parallel and perpendicular lines, properties of polygons)
- demonstrates understanding of area and volume formulas and Cavalieri's principle, and uses them to model and solve problems
- identifies relationships between two-dimensional and three-dimensional objects
- applies geometric concepts in modeling situations (e.g., using shapes to describe objects, applying concepts of density based on area and volume)
- demonstrates knowledge of non-Euclidean geometry

Geometry and Measurement Sample Question

GEOMETRY AND MEASUREMENT SAMPLE QUESTION

5. Use the diagram below to answer the question that follows.



Given:

$\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{DC}$

$\overleftrightarrow{BC}$ is parallel to $\overleftrightarrow{AD}$

Prove:

$\triangle ABC \cong \triangle CDA$

The student reasons as follows: "Draw a line between point A and point C. Then, $\angle BCA \cong \angle DAC$. Also note that $\angle BAC \cong \angle DCA$. Now, $\overline{AC} \cong_{co} \overline{AC}$ so the two triangles are congruent by angle-side-angle."

Geometry and Measurement Sample Question

Which statement best justifies the second and third sentences in the student's response?

a. If two lines are cut by a transversal and same-side interior angles are congruent, then the lines are parallel.

b. two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

c. If two parallel lines are cut by a transversal, then same-side interior angles are congruent.

d. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Pedagogical Content Knowledge

- identifies the skills and conceptual understanding necessary for students to achieve a specific new learning goal
- demonstrates knowledge of methods for assessing student readiness for a specific new learning goal
- demonstrates knowledge of ways to connect students' prior learning to the new learning goal
- promotes coherence by connecting learning across the mathematical domains
- describes an appropriate and effective instructional strategy that includes multiple representations of essential/difficult concepts
- demonstrates knowledge of methods for assessing students' progress during the lesson toward achieving the learning goal

PCK SAMPLE QUESTION

Use the information below to complete the task that follows.

As a mathematics teacher, you are preparing to teach a lesson to address an aspect of the following standard from the New York State P–12 Common Core Learning Standards:

> **Interpreting Functions (F–IF)**
> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases, and using technology for more complicated cases.
>
> a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Prepare a response of approximately 400–600 words in which you:

state the student learning objectives for this lesson;

describe the conceptual understanding, skills, and prerequisite knowledge that students need in order to understand the content described by this learning standard;

describe an effective instructional strategy that promotes student understanding of the content described by the learning standard;

describe a method for helping students build a viable argument related to the learning standard; and

describe a method for assessing students' progress toward the goal of understanding the content described by the learning standard.