Abstract

For real x, put

$$((x)) = \begin{cases} x - [x] - 1/2 & \text{if } x \notin \mathbf{Z}, \\ 0 & \text{if } x \in \mathbf{Z}, \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If h and k are positive integers, the classical Dedekind sum is defined by

$$s(h,k) = \sum_{\mu \bmod k} \left(\left(\frac{h\mu}{k}\right)\right)\left(\left(\frac{\mu}{k}\right)\right),$$

where the summation is over a complete residue system modulo k. Dedekind introduced this sum in connection with the transformation formula for the Dedekind eta function and deduced from this his famous reciprocity formula

$$s(h,k) + s(k,h) = -\frac{1}{4} + \frac{1}{12}\left(\frac{h}{k} + \frac{1}{hk} + \frac{k}{h}\right) \qquad (\gcd(h,k) = 1).$$

In this talk, we give an elementary proof of this formula from a purely arithmetic point of view.