Math 2675: Calculus III

Sample Questions 4: Lines and Planes **Professor:** Caner Koca

1. Find the line passing through $(1, -2, 6)$ and parallel to the vector $\langle 5, 2, -3 \rangle$.
Solution: Recall that the line equation is given as:

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

or for short

$$\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle.$$

Thus, the equation of the line is

$$\begin{aligned} x &= 1 + 5t \\ y &= -2 + 2t \\ z &= 6 - 3t \end{aligned}$$

Note that the components of the direction vector become the *coefficients* of the t-parameter. Alternatively, you can write the equations as:

$$\vec{r}(t) = \langle 1 + 5t, -2 + 2t, 6 - 3t \rangle.$$

2. Find the intersection of this line (described above) with the plane $3x - 2y + z = 29$.
Solution. At the point of intersection of the line with the plane, the x, y, and z coordinates are the same. Thus we plug in the x, y, and z coordinates of the line

$$\begin{aligned} x &= 1 + 5t \\ y &= -2 + 2t \\ z &= 6 - 3t \end{aligned}$$

in the plane equation $3x - 2y + z = 29$. We get

$$\begin{aligned} 3x - 2y + z &= 10 \\ 3(1 + 5t) - 2(-2 + 2t) + (6 - 3t) &= 10 \\ 3 + 15t + 4 - 4t + 6 - 3t &= 29 \\ 8t &= 16 \\ t &= 2. \end{aligned}$$

This means, the intersection occurs on the line when the time parameter t equals 2. Thus the intersection point is:

$$\begin{aligned} x &= 1 + 5t = 1 + 5 \cdot 2 = 11 \\ y &= -2 + 2t = -2 + 2 \cdot 2 = 2 \\ z &= 6 - 3t = 6 - 3 \cdot 2 = 0. \end{aligned}$$

That is, the intersection point of the line with the given plane is $(x, y, z) = (11, 2, 0)$.