Math 2675: Calculus III

Sample Questions 2: Vectors **Professor:** Caner Koca

1. Write down the equation of the sphere of radius 3, centered at $(-1, 3, 0)$.
   Answer: $(x-(-1))^2 + (y-3)^2 + (z-0)^2 = 3^2$, or if you simplify $(x+1)^2 + (y-3)^2 + z^2 = 9$

2. Find the rectengular coordinates (x, y, z) of a point which has spherical coordinates $(\rho, \theta, \varphi) = (3, \pi/3, \pi/2)$.
   Solution: We have seen in class that

$$x = \rho \cos\theta \sin\varphi$$
$$y = \rho \sin\theta \sin\varphi$$
$$z = \rho \cos\varphi$$

   Hence

$$x = 3\cos\pi/3 \sin\pi/2 = 3 \cdot 1/2 \cdot 1 = 3/2$$
$$y = 3\sin\pi/3 \sin\pi/2 = 3 \cdot \sqrt{3}/2 \cdot 1 = 3\sqrt{3}/2$$
$$z = 3\cos\pi/2 = 3 \cdot 0 = 0$$

   So we get the point $(3/2, 3\sqrt{3}/2, 0)$. Note that it's a point on the xy-plane.

3. Find the length and the direction vector of $\langle 1, 2, 3\rangle$.
   Solution: $\|\langle 1, 2, 3\rangle\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$.
   $\operatorname{dir}(\langle 1, 2, 3\rangle) = \frac{1}{\sqrt{14}}\langle 1, 2, 3\rangle = \langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14}\rangle$