Fall 2012 MAT 2580 Review Sheet

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Here are some practice questions that should look familiar to all of you. Please practice using row operations on your calculator for each exercises that requires you show individual steps. You will not be allowed to use the det command on your calculator for the midterm.

1. For

$$A = \begin{bmatrix} 13 & 24 & 56 & 78 \\ 91 & -2 & -34 & -56 \\ -78 & -89 & 10 & 23 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & -1 & -2 & -3 \\ -2 & -1 & 0 & 1 \\ 2 & 3 & 4 & 5 \end{bmatrix},$$

   add the following, explaining why a sum is undefined if it is so:

   a. $A + 0_{3,4}$,

   b. $B - 4I_4$, and

   c. $A + B^T$.

2. Multiply the following:

   a. $2\begin{bmatrix} 1 & 0 & 2 \\ 4 & 3 & 0 \\ 0 & -4 & -1 \end{bmatrix}$,

   b. $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} I_2$,

   c. $\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & b \\ 0 & 3 \end{bmatrix}$,

   d. $\begin{bmatrix} 1 & b \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & b \\ 0 & 3 \end{bmatrix}$, and

   e. $\begin{bmatrix} 1 & b \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & b \\ -1 & 3 \end{bmatrix}$.

3. Use elemetary row operations to complete the forward elimination of the row reduction process on this matrix to produce a matrix in row echelon form then circle the pivot positions in the matrix on this page:

$$\begin{bmatrix} 0 & 1 & 2 & 4 \\ 3 & -2 & -1 & -5 \\ -3 & 1 & 0 & 0 \end{bmatrix}.$$

4. Please circle the pivot positions in the matrix below. Then use elemetary row operations to complete the backward elimination of the row reduction process on this matrix to produce a matrix in row echelon form:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -2 & -1 & -5 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

5. Please construct the 4×4 elementary matrix for the operation that swaps the second and fourth rows of a matrix when multiplying it on the left.

6. If a 4×5 matrix has pivot positions in the positions $(1, 1)$, $(2, 3)$, and $(3, 4)$, how many solutions does the system of linear equations it represents have?

7. Answer each of the following:

   (a) In terms of pivot positions of A, when does the equation $A\mathbf{x} = \mathbf{0}$ have only one solution?

   (b) In terms of pivot positions of A, when does the equation $A\mathbf{x} = \mathbf{b}$ always have solution for any $\mathbf{b}$?

   (c) In terms of pivot positions of A, when are the columns of A linearly independent?

   (d) In terms of pivot positions of $A : n \times m$, when do the columns of A span $\mathbb{R}^n$?

8. Solve the equation $A\vec{x} = \vec{0}$ for A below. Do not use the `rref`() operator or the x^{-1} operator on your calculator to do this. List each step. Write your answer in parametric vector form.

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 & 5 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}.$$

9. The function from $\mathbb{R}^3$ to $\mathbb{R}^3$ which sends each vector to its closest vector on the plane $x_1 + x_2 + x_3 = 0$ is a linear transformation. Construct the standard matrix for this linear transformation. This function is defined by the rule:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \frac{1}{3} \begin{bmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{bmatrix}.$$

10. Suppose that the parametric vector solution to the homogeneous equation $C\vec{x} = \vec{0}$ is $x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ and that $C \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Solve $C\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

11. Use `rref`() to determine whether the map $\mathrm{T}_D(\vec{x}) = D\vec{x}$ surjective (onto). Here

$$D = \begin{bmatrix} 1 & -1 & 2 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & -2 & 1 \end{bmatrix}.$$

12. Use `rref`() to determine whether the columns of J linearly independent.

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

13. Use `rref()` to find the inverse of F.

$$F = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & 0 & 1 \\ 0 & -3 & 2 & 0 \\ 3 & 1 & 5 & 1 \end{bmatrix}.$$

14. Suppose G is square and $\texttt{rref}([G|I]) = [K|L]$ where K is square but $K \neq I$. Why is G not invertible?

15. Find the pivot positions is matrix H below by performing forward elimination. Circle the pivot positions in H. Show each step. Use neither the `rref()` command nor the x^{-1} command when doing this.

$$H = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \end{bmatrix}.$$

16. Perform forward elimination on A to construct a matrix row equivalent to A in row echelon form. Use this to determine the rank of A. Show each step of the elimination.

$$A = \begin{bmatrix} 0 & 1 & 3 & -1 & 4 \\ 3 & 1 & 2 & -4 & 2 \\ -4 & 1 & 1 & 4 & 2 \\ 2 & 1 & 1 & 0 & 2 \end{bmatrix}$$

17. Perform backward elimination on B to construct a matrix row equivalent to B in row echelon form. Use this to construct a basis for the null space of B. Show each step of the elimination.

$$B = \begin{bmatrix} 1 & 2 & 4 & -2 & -1 \\ 0 & 1 & 2 & -4 & -2 \\ 0 & 0 & 0 & -4 & 2 \end{bmatrix}$$

18. Suppose that $C : 9 \times 6$ has rank 5. What is the dimension of the null space of C?

19. Use `rref` to find a basis for the column space of D.

$$D = \begin{bmatrix} 0 & 2 & 3 & 0 & 2 \\ -3 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ -3 & 2 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

20. A subspace S of $\mathbb{R}^5$ has dimension 4 and contains $\vec{v}_1$,$\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$, which are linearly independent. Explain why the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is S.

21. F^{-1} and $\vec{b}$ are given below. Use this information and matrix multiplication to solve $F\vec{x} = \vec{b}$.

$$F^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 3 & -2 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

22. Use `rref` to compute the nullity of G

$$G = \begin{bmatrix} 1 & 2 & 5 & -4 & 5 \\ -3 & 2 & 1 & -4 & 5 \\ 2 & 3 & 8 & -6 & -5 \end{bmatrix}$$

23. Use the definition of subspace to explain why the set of vectors with at most one non-zero coordinate is not a subspace of $\mathbb{R}^3$?

24. Using forward elimination without scaling please compute the determinant:

$$\begin{vmatrix} 1 & 2 & -1 & -1 \\ 2 & 5 & -2 & 1 \\ 0 & -3 & 0 & 1 \\ 3 & 1 & -5 & 1 \end{vmatrix}.$$

25. Compute the inverse of

$$\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

26. Compute the determinant of

$$\begin{bmatrix} 2 & -3 & 0 \\ 4 & 0 & 4 \\ 1 & -1 & 5 \end{bmatrix}$$

27. Use cofactor expansion to compute the determinant of

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 4 & 0 & 4 \\ 0 & 1 & -1 & 5 \\ -1 & 2 & 0 & 1 \end{bmatrix}.$$

28. Showing each step, compute the determinant using a mixed approach:

$$\begin{bmatrix} 1 & 2 & -3 & 0 & -1 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 1 & -1 & 5 & 0 \\ -1 & 2 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}.$$