## Interpreting Pivot Positions

The tables below organize our understanding of pivot positions of a matrix as to how they convey information about the structure that the matrix represents.

|  | Information that matrix represents |  |  |
| ---: | :--- | :--- | :--- |
| Pivot Positions are in | A Coefficient Matrix | A as list of column n-vectors | The Linear transformation $T_{A}$ |
| Every row of $A$ | A solution to $A x=b$ always exists. | The columns of $A$ span $R^{n}$. | $T_{A}$ is onto. |
| Every column of $A$ | Any solution to $A x=b$ is unique. | The columns of $A$ are linearly <br> independent. | $T_{A}$ is 1-to-1. |

Here is a supplementary table which addresses the role of pivot positions in the augmented matrix of an equation.

| Augmented Matrix [Alb] of the equation Ax=b |  |
| :---: | :---: |
| A pivot position is | If and only if |
| In the right-most column | There is no solution to $\mathrm{Ax}=\mathrm{b}$. |
| In the coefficient matrix | The variable of that column is basic. |
| Missing from the right-most column | There is a solution to $A x=b$. |
| Missing from a column of the coefficient matrix | The variable of that column is free. |

