Linear Algebra
9/6/2012

Topics: Row operations, elementary matrices and linear equations

NOTE: This lesson follows a lecture on Linear combinations and Linear independence.

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RECALL

A LINEAR COMBINATION OF x_1, x_2, x_3 IS OF THE FORM $a_1x_1 + a_2x_2 + a_3x_3$ FOR $a_i \in \mathbb{R}$. IN OTHER WORDS IT IS OF THE FORM

x_i: INDETERMINATE (VECTORS OR NUMBER OR WHATEVER MAKES SENSE)

$$[x_1\ x_2\ x_3]\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}.$$

$\leftarrow$ WEIGHTS

$\uparrow$ POSSIBLY COLUMN VECTORS

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A SET OF VECTORS $\{\vec{V_1}, \vec{V_2}, \vec{V_3}\}$

IS LINEARLY INDEPENDENT

IFF $a_1\vec{V_1} + a_2\vec{V_2} + a_3\vec{V_3} = \vec{0}$

IMPLIES

$a_1 = a_2 = a_3 = 0$

LATER

THAT IS THE (HOMOGENEOUS) EQUATION

$$\begin{bmatrix} \vec{V_1} & \vec{V_2} & \vec{V_3} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \vec{0}$$

ONLY HAS A TRIVIAL SOLUTION

NOTE: THIS MEANS NO $\vec{V_i}$ IS A LINEAR COMBINATION OF THE OTHER TWO.

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HERE THIS MEANS

$\vec{V_1}, \vec{V_2}, \vec{V_3}$ ARE NOT COPLANAR.

THIS GENERALIZES. TO n VECTORS NOT SHARING n-1 DIMENSION SPACE IFF THEY ARE LINEARLY INDEPENDENT

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MULTIPLICATION ON THE LEFT IS A ROW OPERATION.

ELEMENTARY OPERATIONS (SECT 1.1)

EXAMPLES OF THE CORRESPONDING ELEMENTARY MATRICES (SECT 2.2)

- SWAP
- REPLACE
- SCALE

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ x \\ z \end{bmatrix}$$ SWAP ROWS 1 & 2.

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -2x+y \\ z \end{bmatrix}$$ REPLACE 2^{nd} ROW WITH ITS SUM WITH -2 TIMES THE 1^{st}.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 3z \end{bmatrix}$$ SCALE 3^{rd} ROW BY 3.

NOTE FROM SECT. 2.2 THE RELATIONSHIP BETWEEN ELEMENTARY MATRICES AND I_3.

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CONSIDER

COEFFICIENT MATRIX

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 3 & 7 \\ 1 & 5 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$ MATRIX EQUATION

IT MEANS

$$\left.\begin{aligned} x_1 - 2x_2 \quad &= -2 \\ 3x_2 + 7x_3 &= 1 \\ x_1 + 5x_2 \quad &= 3 \end{aligned}\right\}$$ SYSTEM OF LIN EQ.

AUGMENTED MATRIX OF THE SYSTEM

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & -2 \\ 0 & 3 & 7 & 1 \\ 1 & 5 & 0 & 3 \end{array}\right]$$

PARTITION

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ROW OPERATION WORK
THE SAME FOR ALL 3.

$$A\vec{v}=\vec{b} \quad \text{vs} \quad [A|\vec{b}]$$

E: ELEMENTARY MATRIX

$$EA\vec{v}=E\vec{b} \quad \text{vs} \quad E[A|\vec{b}]$$
$$=[EA|E\vec{b}]$$

NOTE
IF $\vec{b}=\vec{0}$ THEN THE SYSTEM
IS CALLED HOMOGENEOUS. (SECT 1.5)

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