LINEAR ALGEBRA

9/19/2012

TOPIC: LINEAR TRANSFORMATIONS

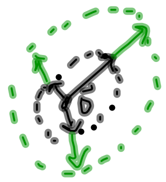
IDEA: MATRICES ARE LINEAR TRANSFORMATIONS

Sep 19-9:21 AM

CONSIDER THE TRANSFORMATIONS FROM GEOMETRY: (IN $\mathbb{R}^2$)

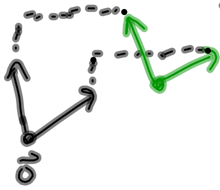
DILATION (BY r)

$\vec{v} \mapsto D_r \vec{v}$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} rx \\ ry \end{bmatrix}$



TRASLATION (by (a,b))

$\vec{v} \mapsto T_{a,b} \vec{v}$



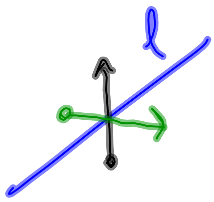
$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+a \\ y+b \end{bmatrix}$

ROTATION (BY θ)

$\vec{v} \mapsto r_\theta \vec{v}$

REFLECTION (ACROSS ℓ)

$\vec{v} \mapsto R_\ell \vec{v}$



SHEAR (by n)

$\begin{bmatrix} x \\ y \end{bmatrix} \longmapsto \begin{bmatrix} x \\ y+nx \end{bmatrix}$

WHICH OF THESE ARE LINEAR TRANSFORMATIONS OR MATRICES?

Sep 19-9:23 AM

NOTICE $D_r \vec{v} = \begin{bmatrix} 1 & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} r & 0 \\ 0 & 1 \end{bmatrix} \vec{v} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \vec{v}$

SCALAR ELEMENTARY MATRICES.

ALSO (A NON-TRIVIAL) TRASLATION CANNOT BE A LINEAR TRANSFORMATION OF A MATRIX SINCE

$T_{a,b} \vec{0} = \begin{bmatrix} a \\ b \end{bmatrix}$ AND $A\vec{0} = \vec{0}$

Sep 19-9:32 AM

WE DEFINE LINEAR TRANSFORMATIONS L.T. (INDEPENDENT OF MATRICES)
BY THE FOLLOWING TWO PROPERTIES.

$$T(\vec{v}+\vec{u}) = T(\vec{v}) + T(\vec{u})$$

$$T(r\vec{v}) = rT(\vec{v})$$

(pg 65)
COMPARE TO
THM. 5 pg 39

EVERY L.T. OF
A MATRIX SATISFIES
THIS DEFINITION

WE WANT TO BUILD
A MATRIX FROM A LINEAR
TRANSFORMATION, S.T. THAT
L.T. IS THE L.T. OF THAT
MATRIX.

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NOTE $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\left(x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$= xT\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + yT\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

SO THE VALUES $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ & $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

DETERMINE T.

SET $A = \left[T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \; T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \right]$ THE T IS THE L.T. OF A.

1st COLUMN OF I_2 2nd COLUMN OF I_2

SEE THM 10 PG. 71

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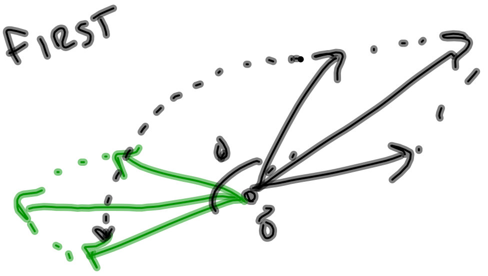
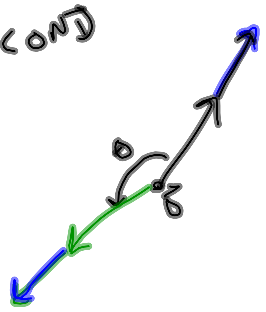
ROTATION IS A L.T.

FIRST



SECOND



ADDITION

$$r_\theta(\vec{v}+\vec{u}) = r_\theta(\vec{v}) + r_\theta(\vec{u})$$

MULTIPLICATION

$$r_\theta(s\vec{v}) = s\, r_\theta(\vec{v})$$

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