Linear Algebra
9/12/2012

Topics: Row echelon form, pivot positions, systems of linear equations and linear independence.

Idea: Pivot positions are a property of the matrix as a datum, which reflect on the information it represents.

Question: I want a degree 3 polynomial in one variable which passes through the following points:

(-2,-3), (-1,2), (1,2), (2,4)

Suggestions:

Draw a curve through them.
I want coefficients of my polynomial

If there were two points and the degree were 1 we could find the line. (linear equation)
OK. Here is a new way to do that...

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POINTS

$(-2,-3), (-1,2)$ $mx+b=y$

$$\begin{matrix} m(-2)+b=-3 \\ m(-1)+b=2 \end{matrix} \Rightarrow \begin{bmatrix} -2 & 1 & -3 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\sim_{\text{SWAP}} \begin{bmatrix} -1 & 1 & 2 \\ -2 & 1 & -3 \end{bmatrix} \sim_{\text{REPLACE}} \begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & -7 \end{bmatrix}$$

$$\sim_{\text{REPLACE}} \begin{bmatrix} -1 & 0 & -5 \\ 0 & -1 & -7 \end{bmatrix} \sim_{\text{SCALE SCALE}} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \end{bmatrix} \text{ SOLVED}$$

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POLYNOMIAL QUESTION REMAINS OPEN. (NEXT TIME)

IN SOLVING A SYSTEM OF LINEAR EQUATIONS WE SHOULD BE USING ELIMINATION IN THIS CLASS

EXAMPLE: (pg 15 – 17) GAUSS, JORDAN

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

FORWARD PHASE

$$\sim \begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

ROW ECHELON FORM (ref)

BACKWARD PHASE

$$\sim \begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

REDUCED ROW ECHELON FORM (rref)

SEE PG 13

SOLVED (THIS SOLUTION LETS x_3 AND x_4 VARY FREELY)

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ANOTHER EXAMPLE IN ROW ECHELON FORM

$$\begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

WE KNOW THERE IS NO SOLUTION.

ANOTHER EXAMPLE

$$\begin{bmatrix} 3 & -2 & 1 & 0 \\ 0 & 1 & -4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

WE KNOW x_3 VARIES FREELY AND THAT THERE IS A UNIQUE SOLUTION FOR EACH VALUE OF x_3.

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NOTE: (REDUCED) ROW ECHELON FORM DESCRIBES MATRICES INDEPENDENT OF THEIR MEANING.

DEFINITION: IF B IS COMPUTABLE FROM A BY A SEQUENCE OF ELEMENTARY ROW OPERATIONS (THAT IS MULTIPLICATION ON THE LEFT BY A SEQUENCE OF ELEMENTARY MATRICES) WE SAY B IS ROW EQUIVALENT TO A. $(B \sim A)$

ROW ECHELON FORM REVEALS THE FOLLOWING STRUCTURE

FACT: THE LOCATION OF THE LEADING ENTRIES OF A ref MATRIX $\sim A$, IS INDEPENDENT OF WHICH ROW OPERATIONS WERE IN THE SEQUENCE.
SUCH LOCATIONS ARE CALLED THE PIVOT POSITIONS OF A.

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EXAMPLE:

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \leftarrow \text{PIVOT POSITIONS}$$

$$\sim \begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

FACTS ABOUT PIVOT POSITIONS IN AUGMENTED MATRICES (THM 2)

- THE SYSTEM HAS NO SOLUTION IFF THE AUGMENTED COLUMN CONTAINS A PIVOT POSITION
- ANY SOLUTION IS UNIQUE IFF EVERY OTHER COLUMN HAS A PIVOT POSITION.

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COROLLARY:

THE COLUMNS OF A ARE LINEARLY INDEPENDENT
IFF
EACH COLUMN CONTAINS A PIVOT POSITION.

SEE AUGMENTED MATRIX $[A \mid \vec{0}]$

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