Linear Algebra
9/10/2012

Topic: Linear combinations, Linear independence.

Also: algebraic properties of transpose and partitions.

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A partitioned matrix is a matrix whose columns and rows are grouped together for organization.

(SEE SECT 2.4)

EXAMPLE

$$A = \left[\begin{array}{cc|ccc} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ \hline 5 & 6 & 1 & 0 & 0 \\ 0 & 7 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \leftarrow 1^{st} \\ \\ \leftarrow 2^{nd} \end{array}$$

1st COLLECTION OF COLUMNS

2nd

CONSIDER (CONFIRM PROPERTIES ON NEXT PAGE)

$$B = \left[\begin{array}{cc} 1 & 2 \\ 0 & 1 \\ \hline 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{array}\right] \quad \text{AND } A \cdot B.$$

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Properties of Transposition: A,B and C are matrices and r is a real number.

$A^T+B^T=(A+B)^T$

$rA^T=(rA)^T$

$(A^T)^T=A$

$C^TA^T=(AC)^T$

(SEE SECT 2.4)

Properties of Partitioned matrices: ASSUME PARTITIONS ARE 'COMPATIBLE'.

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] + \left[\begin{array}{c|c} E & F \\ \hline G & H \end{array}\right] = \left[\begin{array}{c|c} A+E & B+F \\ \hline C+G & D+H \end{array}\right]$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \cdot \left[\begin{array}{c|c} E & F \\ \hline G & H \end{array}\right] = \left[\begin{array}{c|c} AE+BG & AF+BH \\ \hline CE+DG & CF+DH \end{array}\right]$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]^T = \left[\begin{array}{c|c} A^T & C^T \\ \hline B^T & D^T \end{array}\right]$$

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$$A = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 5 & 6 & 1 & 0 & 0 \\ 0 & 7 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad A \cdot B.$$

SHORTCUT

$$A = \left[\begin{array}{c|c|c} I_2 & \begin{matrix} 2 & 3 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 \\ -1 \end{matrix} \\ \hline \begin{matrix} 5 & 6 \\ 0 & 7 \end{matrix} & I_2 & O_{2\times 1} \end{array}\right] \qquad B = \left[\begin{array}{c} \begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \\ \hline I_2 \\ \hline O_{1\times 2} \end{array}\right]$$

IDENTITIES

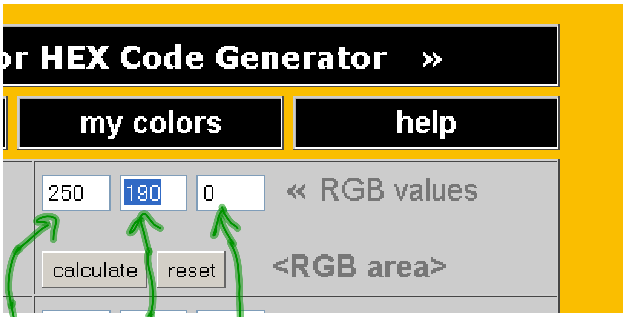
ZERO

$$A \cdot B \left[\begin{array}{c} I_2 \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} I_2 + 0 \\ \hline \begin{bmatrix} 5 & 6 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + I_2 + 0 \end{array}\right] = \left[\begin{array}{c} \begin{matrix} 3 & 5 \\ 0 & 1 \end{matrix} \\ \hline \begin{bmatrix} 5\cdot 1+0 & 5\cdot 2+6\cdot 1 \\ 0+0 & 0+7\cdot 1 \end{bmatrix} + I_2 \end{array}\right] = \left[\begin{array}{cc} 3 & 5 \\ 0 & 1 \\ \hline 6 & 16 \\ 0 & 8 \end{array}\right]$$

LINEAR COMBINATIONS

NOTICE THIS EXAMPLE AT

http://drpeterjones.com/colorcalc/



AMOUNTS OF RED GREEN & BLUE RESP.

A LINEAR COMBINATION IS A SUM OF SCALARS OF "THINGS"*

HERE WE HAVE THE LINEAR COMBINATION

$250r + 190g + 0b$

NOTE: THIS CAN BE NATURALLY REPRESENTED BY THE VECTOR $\begin{bmatrix} 250 \\ 190 \\ 0 \end{bmatrix}$

* "THINGS" ARE IN THIS CASE ARE "ABSTRACT VECTORS" (SEE CH 4)

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NOTE: COLUMNS OF I_3

$$\begin{bmatrix} 250 \\ 190 \\ 0 \end{bmatrix} = 250\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 190\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{SO}$$

VECTORS ARE LINEAR COMBINATIONS.

FURTHER A LINEAR COMBINATION OF $\underset{}{n}$-VECTORS IS AN n-VECTOR. (# OF ENTRIES)

RECALL GARRY'S HW, (SECT 2.1) SAY $\vec{v}$ IS A 3-VECTOR $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

$$\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} \vec{v} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v_1\vec{a}_1 + v_2\vec{a}_2 + v_3\vec{a}_3$$

SO MATRIX MULTIPLICATION OF A VECTOR RESULTS IN A LINEAR COMBINATION OF ITS COLUMNS.

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WE SAY A SET OF VECTORS $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$
IS LINEARLY DEPENDENT IF

HOMOGENEOUS EQUATION SECT 1.5

THERE IS A SOLUTION TO

$$\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix} \vec{x} = \vec{0}$$

NONTRIVIAL SOLUTION SECT. 1.5

HAS A SOL'N WHERE $\vec{x} \neq 0$

OTHERWISE $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IS CALLED LINEARLY INDEPENDENT, (AND THE ARROWS OF $\vec{v}_1, \vec{v}_2, \vec{v}_3$ ARE NOT COPLANAR)

(SEE SECT 1.7)

BECAUSE NO $\vec{v}_i$ IS A LINEAR COMBINATION OF THE OTHER

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