Linear Algebra
9/5/2012

Topic: Row Operations and Equations.

Remember: Elementary Matrices perform row operations.

Sep 5-8:39 AM

Homework Questions:

HERE D IS CALLED A "DIAGONAL MATRIX"

pg.100: 11

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$DA = \begin{bmatrix} 5 & 10 & 15 \\ 6 & 12 & 15 \\ 6 & 10 & 12 \end{bmatrix} \qquad AD = \begin{bmatrix} 5 & 6 & 6 \\ 10 & 12 & 10 \\ 15 & 15 & 12 \end{bmatrix}$$

NOTE: $A^T = A$ WE SAY A IS <u>SYMMETRIC</u>.

Sep 5-8:45 AM

GOAL: FIND B S.T. $AB = BA$

NOTE

$2 \cdot 5$ VS. $2 \cdot 3$

$$DA = \begin{bmatrix} 5 & 10 & 15 \\ 6 & 12 & 15 \\ 6 & 10 & 12 \end{bmatrix} \qquad AD = \begin{bmatrix} 5 & 6 & 6 \\ 10 & 12 & 10 \\ 15 & 15 & 12 \end{bmatrix}$$

SUGGESTION
TRY MAKING DIAGONAL ENTRIES THE SAME

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

E.G. $B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

I.E. $B = 5 \cdot I_3$

$BA = 5A = AB$

CHECK IT OUT

$A \cdot A^2 = A^2 \cdot A$

SO $B = A^2$

CHECK THIS TO BE SURE

Sep 5-8:54 AM

CONSIDER THE FOLLOWING PRODUCTS

$$\underset{A}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{\vec{v}}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{\vec{u}}{\begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix}}$$

__SWAP__ 1^{ST} ~~ENTRY~~ ROW WITH THE 2^{nd} (TRANSPOSED ENTRIES)

HERE, A IS A SWAP MATRIX,

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_2 \\ x_3 \end{bmatrix}$$

__REPLACE__ 1^{ST} ROW WITH THE SUM OF ITSELF WITH -2 TIMES THE 2^{nd}.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ 3 \cdot x_3 \end{bmatrix}$$

__SCALE__ THE 3^{rd} ROW BY 3

Sep 5-9:15 AM

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NEW 1ST ROW IS 1 TIMES OLD FIRST ROW MINUS 2 TIME SECOND OLD ROW

2nd ROW SAME

3rd ROW SAME

NOTE: THIS INTERPRETATION DOES NOT DEPEND ON THE # OF COLUMNS

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix} = \begin{bmatrix} x_{11}-2x_{21} & x_{12}-2x_{22} & x_{13}-2x_{23} & x_{14}-2x_{24} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix}$$

THE PREVIOUS EXAMPLES ARE CALLED ELEMENTARY MATRICES. SEE SECT. 2.2

FOR THEIR ROW OPERATIONS SEE SECT 1.1

Sep 5-9:26 AM

CONSIDER

(CONTRAST TO PG 5 EXAMPLE 1)

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}$$

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -9 \end{bmatrix}$$

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned}$$

HERE $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -9 \end{bmatrix}$$

$\downarrow E_1 \cdot (-)$ $\qquad \downarrow E_1 \cdot (-)$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 0 & -3 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -9 \end{bmatrix}$$

HERE A IS A COEFFICIENT MATRIX

THIS IS CALLED AN AUGMENT MATRIX →

PARTITION ↓

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{array}\right]$$

$\downarrow E_1 \cdot (-)$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & -3 & 13 & -9 \end{array}\right]$$

Sep 5-9:33 AM

Sep 5-9:41 AM