Linear Algebra
8/29/2012

Topic: More on Operations of matrices and some uses of matrices.

Thought: A matrix can be interpreted as a function.

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Some Laws of Matrix operations.

See section 2.1 of the book.

A, B and C are matrices. r and s are real numbers.
Assume for each equality that the matrix operations are defined.

Scalar Multiplication:
rA multiplies all of the entries of A by r.

EXAMPLE

$$-3\begin{bmatrix}1\\0\\-2\end{bmatrix}=\begin{bmatrix}(-3)\cdot 1\\(-3)\cdot 0\\(-3)(-2)\end{bmatrix}=\begin{bmatrix}-3\\0\\6\end{bmatrix}$$

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Addition of matrices:
Commutativity: A+B=B+A
Associativity: A+(B+C)=(A+B)+C
Distribution of Scalars: r(A+B)=rA+rB
Additive Identity 0+A=A

HERE: (SUPPOSE $A = 5\times 3$)

$$0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ ALSO DENOTED $0_{5\times 3}$

Additive inverse: A+(-1)A=A+(-A)=A-A=0

COMMENT IF $A: n\times n$ WE SAY A IS SQUARE.

NOTATION: IF A IS SQUARE THEN A^n (n: NATURAL NUMBER) IS $\underbrace{A\cdot A\cdot A\cdots A}_{n \text{ TIMES}} = A^n$ $A^0 = I_n$

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Matrix Multiplication:

Associativity: A(BC)=(AB)C

Scalar compatibility: r(AB)=(rA)B=A(rB)

Multiplicative Identity IA=A

ASSUME $A: n \times m$ THEN $I: n \times n$ SAY $n=3$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \text{ SOMETIMES } 1_3$$

Distribution rule

A(B+C)=AB+ AC

NOTE:
There is no rule of commutativity of matrix multiplication.
Matrices do not necessarily have multiplicative inverses.
AB=0 does NOT imply A=0 or B=0.

EXAMPLE

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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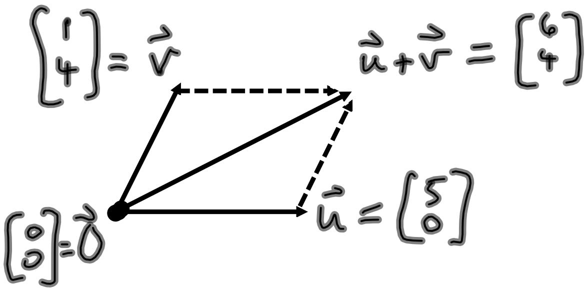
Some special matrices:

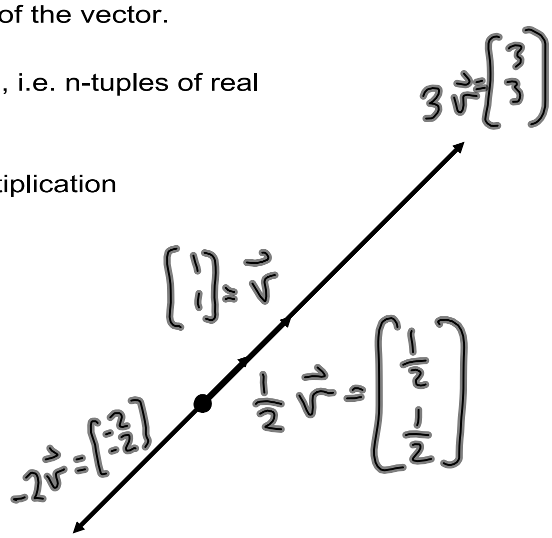
If a matrix has one column (one row) we call such a matrix a column- (resp. row-) vector. The number of rows (columns) in a column-vector (row-) is called the size of the vector.

We can think of size n column-vectors as arrows in $\mathbf{R}^n$ from the origin to the point whose coordinates match that of the vector.

R: real number line $\mathbf{R}^n$:n-dimensional space, i.e. n-tuples of real numbers.

We can reconsider addition and scalar multiplication





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Note: If three vectors are coplanar in the sense of their arrows their sum and sum of any scalars of them will also sit on the same plane.

Now I can consider multiplying a vector by a matrix on the right.

If A:mxn matrix and **v** is an n-vector (size n vector), then A**v** is an m-vector. In this sense an mxn matrix can be thought of as a function from $\mathbf{R}^n$ to $\mathbf{R}^m$. This function **v** |-> A**v** is called the linear transformation of A. Denote $T\text{-}_A(\mathbf{v})=A\mathbf{v}$. Such functions can be described by $\mathbf{R}^n \text{->} \mathbf{R}^m$.

We will see more of $T\text{-}_A(\mathbf{v})$ after test 1.

If A:mxn matrix and **v** is an n-vector and **u** is an m-vector, then $\mathbf{u}^TA\mathbf{v}$ is a 1-vector, that is, a real number. In this sense an mxn matrix can be thought of as a function from $\mathbf{R}^n x\mathbf{R}^m$ to **R** (Here $\mathbf{R}^n x\mathbf{R}^m$ means pairs, the left element is in $\mathbf{R}^n$ and the right is in $\mathbf{R}^m$). Such functions can be denoted as $\mathbf{R}^n x\mathbf{R}^m \text{->} \mathbf{R}$. Denote $B_A(v,u)=\mathbf{u}^TA\mathbf{v}$.

We will see more of $B_A(v,u)=\mathbf{u}^TA\mathbf{v}$ near the end of the semester.

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