Linear Algebra
Sect 1594
8/28/2012

Topic: Matrix Operations

Thought: A matrix is a datum.

'Datum' is the singular form of data.

Aug 28-6:32 PM

A matrix is a (two-dimensional) array of (real) numbers that is organized into rows and columns.

<u>EXAMPLE</u>

$$\begin{array}{rc} & \text{2nd column} \\ \text{2nd Row} & \underbrace{\begin{bmatrix} 1 & 0 & -1 & 3 \\ -2 & -3 & 4 & 5 \end{bmatrix}}_{\text{4 columns}} \left.\vphantom{\begin{matrix}1\\1\end{matrix}}\right\} \text{2 rows} \end{array}$$

The size of a matrix is the number of rows crossed with then number of columns. This example has size 2x4.

Aug 28-6:40 PM

The (i,j) entry of a matrix is the number in the i-row and the j-collumn.

$$\begin{bmatrix} 1 & 0 & -1 & 3 \\ -2 & -3 & 4 & 5 \end{bmatrix}$$

Here the (2,3) entry is 4.

Thought: When thinking about matrices as data, we distinguish data as "information without context".
We will see some contexts next time.

Now we will study basic operations on these data:
Addition, Multiplication and Transposition

Aug 28-6:46 PM

Addition of Matrices

If two matrices have the same size, then their sum exists and is also of that size. Otherwise the sum of two matrices is undefined.

The (i,j) entry of the sum of two matrices is the sum of the respective (i,j) entries of the summands.

EXAMPLES

$$\underset{2\times 2}{\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}} + \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 3+0 & 4+1 \\ -1+1 & 2-2 \end{bmatrix}} = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$$

$$\underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} + \underset{1\times 3}{\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}} \text{ IS UNDEFINED.}$$

Aug 28-6:52 PM

Matrix Multiplication

If the matrix on the left has the same number of columns as the matrix on the right has rows, then the product exists. Otherwise it is undefined.

The product of a l x m matrix with a m x n matrix has size l x n. Its (i,j) entry is the sum of the products of the (i,k) entry from the left with the (k,j) entry from the right, for all columns k on the left.

EXAMPLE

STEP 1

STEP 3

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & 5 \\ 6 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 13 & -1 & 5 \\ 0 & 0 & -13 \end{bmatrix}$$

STEP 2

$$= \begin{bmatrix} 1\cdot 3+0\cdot 6 & 1\cdot(-1)+0\cdot(-2) & 1\cdot 5+0\cdot(-3) \\ (-2)\cdot 3+1\cdot 6 & (-2)(-1)+(1)(-2) & (-2)\cdot 5+1\cdot(-3) \end{bmatrix}$$

Aug 28-6:58 PM

MORE EXAMPLES

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3+0 \\ 0-5 \\ -3+5 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$$

3×2 $\quad 2 \times 1$ $\quad 3 \times 1$

3×1 $\quad 3 \times 1$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

IS UNDEFINED

$1 \neq 3$

Aug 28-7:08 PM

The transpose of a matrix

The tranpose of a n x m matrix is a m x n matrix, whose (i,j) entry are the (j,i) entry of the original matrix.

EXAMPLE

$$\begin{bmatrix} 1 & 0 \\ 2 & -3 \\ 3 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -1 \end{bmatrix}$$

3×2 $\qquad$ 2×3

Aug 28-7:14 PM