LINEAR ALGEBRA

MAT 2580

8/27/2012

SECTION 6642

Aug 27-8:56 AM

MATRICES

A MATRIX IS AN (2-DIMENSIONAL) ARRAY OF NUMBERS (REAL) ORGANIZED INTO ROWS AND COLUMNS.

EXAMPLE

$$\begin{array}{rc} & \text{2nd Column} \\ \text{1st Row} & \left.\underbrace{\begin{bmatrix} 2 & 0 \\ -1 & 4 \\ 3 & -5 \end{bmatrix}}_{\text{2 COLUMNS}}\right\} \text{3 ROWS} \end{array}$$

THE SIZE OF A MATRIX IS THE NUMBER OF ROWS CROSSED WITH THE # OF COLUMNS.

HERE THE SIZE IS 3×2.

Aug 27-8:58 AM

$$\begin{bmatrix} 2 & 0 \\ -1 & 4 \\ 3 & -5 \end{bmatrix}$$

WE REFER TO ENTRIES BY THEIR ROW AND COLUMN (RESP.)

HERE THE (3,1) ENTRY IS 3.

THE (2,2) ENTRY IS 4, ETC.

THOUGHT

A MATRIX IS AN ORGANIZED DATUM.

THAT IS "INFORMATION WITHOUT CONTEXT".

SINGULAR FORM OF DATA.

WE WILL SEE SOME CONTEXTS NEXT TIME.

Aug 27-9:02 AM

ADDITION OF MATRICES

TWO MATRICES MAY BE ADDED TOGETHER IF THEY HAVE THE SAME SIZE.

EXAMPLES

$$\underbrace{\begin{bmatrix} 0 \\ 2 \\ 3 \\ 5 \\ 7 \end{bmatrix} + \begin{bmatrix} 10 \\ 9 \\ 8 \\ 7 \\ 6 \end{bmatrix}}_{\text{BOTH } 5\times 1} \text{ IS DEFINED.}$$

$$\overset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}} + \overset{2\times 3}{\begin{bmatrix} 1 & 3 & 9 \\ 2 & 6 & 18 \end{bmatrix}}$$

IS NOT DEFINED

Aug 27-9:09 AM

MATRIX ADDITION (CONT.)

WHEN ADDING TWO MATRICES THE (i,j) ENTRY OF THE SUM IS THE SUM OF THE (i,j) ENTRIES OF THE SUMMANDS.

EXAMPLE

$$\begin{bmatrix} 0 & 1 \\ 2 & -3 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -7 & 5 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0-7 & 1+5 \\ 2+0 & -3+1 \\ 0+2 & 5+0 \end{bmatrix} = \begin{bmatrix} -7 & 6 \\ 2 & -2 \\ 2 & 5 \end{bmatrix}$$

$(1,1)$ $(2,2)$

Aug 27-9:13 AM

MATRIX MULTIPLICATION

THE PRODUCT OF TWO MATRICES IS DEFINED IF THE # OF ROWS ON THE RIGHT EQUALS THE # OF COLUMNS ON THE LEFT.

EXAMPLES

THE PRODUCT HAS THE SAME # OF ROWS AS THE LEFT AND # OF COLUMNS AS THE RIGHT

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$ IS DEFINED, AND HAS SIZE 3×1.

3×2 2×1

MATCH

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$$ IS NOT DEFINED.

2×1 3×2

$1 \neq 3$

Aug 27-9:17 AM

MATRIX MULTIPLICATION (CONT.)

THE (i,j) ENTRY OF THE PRODUCT IS THE SUM OF THE PRODUCTS OF (i,k) ENTRIES FROM THE LEFT WITH (k,j) ENTRIES FROM THE RIGHT.

EXAMPLE

$(1,1)\cdot(1,1)+(1,2)\cdot(2,1)+(1,3)\cdot(3,1)$

$$\underset{2\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 2 & -3 & 0 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 1\cdot(-2)+0\cdot 0+(-1)\cdot 2 \\ 2\cdot(-2)+(-3)\cdot 0+0\cdot 2 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} -4 \\ -4 \end{bmatrix}}$$

(SIZES)

Aug 27-9:24 AM

Another Example

$2\times2 \qquad 2\times3 \qquad \mapsto 2\times3$

$$\begin{bmatrix}1&0\\-2&1\end{bmatrix}\begin{bmatrix}1&3&-7\\2&4&6\end{bmatrix}=$$

$$\begin{bmatrix}1\cdot1+0\cdot2 & 1\cdot3+0\cdot4 & 1\cdot(-7)+0\cdot6\\(-2)\cdot1+1\cdot2 & (-2)\cdot3+1\cdot4 & (-2)(-7)+1\cdot6\end{bmatrix}=$$

$$\begin{bmatrix}1&3&-7\\0&-2&20\end{bmatrix}\begin{array}{l}\leftarrow \text{1st Row of Right}\\ \leftarrow \text{2nd Row plus } (-2)\times \text{ the 1st Row}\end{array}$$

Aug 27-9:31 AM

TRANSPOSE OF A MATRIX

THE TRANSPOSE OF AN $n \times m$ MATRIX IS AN $m \times n$ MATRIX FOR WHICH THE (i,j) ENTRY IS THE (j,i) ENTRY OF THE ORIGINAL

$$\begin{array}{c} \\ \\ (3,1) \\ \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$(4,2)$ $(1,3)$ $(2,4)$

Aug 27-9:42 AM