Linear Algebra
10/4/2012
Topic: Inverting Matrices

Recall that a linear transformation T_A is bijective if and only if A has a pivot position in every column and every row.
Because bijective means both injective and surjective, and T_A is injective if A has a pivot position in every column and surjective if A has a pivot position in every row.
So if A is invertible rref(A)=I.

Here is an observable fact: $T_A(T_B(\mathbf{v}))=T_{AB}(\mathbf{v})$, because $A(B(\mathbf{v}))=(AB)\mathbf{v}$.
This means that if T_B is the (left/right) inverse of T_A then B is a (left/right) inverse of A, that is BA=I=AB. We say $B=A^{-1}$. In general $B^n=A^{-n}$.

Now recall that $\text{rref}(A)=E_nE_{n-1}...E_2E_1A$. So if A is invertible $E_nE_{n-1}...E_2E_1A=I$ so $E_nE_{n-1}...E_2E_1$ is the inverse of A. So $\text{rref}(A)=A^{-1}A$.

Oct 4-6:04 PM

Recall that A[B|C]=[AB|AC] (section 2.4)

So if A is invertible,

$rref([A|I])=E_nE_{n-1}...E_2E_1[A|I]=A^{-1}[A|I]=[I|A^{-1}]$

FIND A^{-1}

FOR

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & 3 & 4 \\ -1 & -2 & 3 & 4 \\ -1 & -2 & -3 & 4 \end{bmatrix}$$

rref([A|I])

Oct 4-6:48 PM

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>> A

A =

     1     2     3     4
    -1     2     3     4
    -1    -2     3     4
    -1    -2    -3     4

>> P=[A,eye(4)]

P =

     1     2     3     4     1     0     0     0
    -1     2     3     4     0     1     0     0
    -1    -2     3     4     0     0     1     0
    -1    -2    -3     4     0     0     0     1
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Oct 4-7:09 PM

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>> rref(P)

ans =

    1.0000         0         0         0    0.5000   -0.5000         0         0
         0    1.0000         0         0         0    0.2500   -0.2500         0
         0         0    1.0000         0         0         0    0.1667   -0.1667
         0         0         0    1.0000    0.1250         0         0    0.1250

>> B=ans([1,2,3,4],[5,6,7,8])

B =

    0.5000   -0.5000         0         0
         0    0.2500   -0.2500         0
         0         0    0.1667   -0.1667
    0.1250         0         0    0.1250

>>
```

Oct 4-7:09 PM

Oct 4-7:08 PM