

Linear Algebra
10/4/2012
Topic: Inverting Matrices

Recall that a linear transformation T_A is bijective if and only if A has a pivot position in every column and every row. Because bijective means both injective and surjective, and T_A is injective if A has a pivot position in every column and surjective if A has a pivot position in every row. So if A is invertible $\text{rref}(A)=I$.

Here is an observable fact: $T_A(T_B(\mathbf{v}))=T_{AB}(\mathbf{v})$, because $A(B\mathbf{v})=(AB)\mathbf{v}$. This means that if T_B is the (left/right) inverse of T_A then B is a (left/right) inverse of A , that is $BA=I=AB$. We say $B=A^{-1}$. In general $B^n=A^{-n}$.

Now recall that $\text{rref}(A)=E_n E_{n-1} \dots E_2 E_1 A$. So if A is invertible $E_n E_{n-1} \dots E_2 E_1 A=I$ so $E_n E_{n-1} \dots E_2 E_1$ is the inverse of A . So $\text{ref}(A)=A^{-1}A$.

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Recall that $A[B|C]=[AB|AC]$ (section 2.4)

So if A is invertible,

$$\text{rref}([A|I])=E_n E_{n-1} \dots E_2 E_1 [A|I]=A^{-1}[A|I]=[I|A^{-1}]$$

FIND A^{-1}

FOR

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 2 & 3 & 4 \\ -1 & -2 & 3 & 4 \\ -1 & -2 & -3 & 4 \end{bmatrix}$$

$$\text{rref}([A|I])$$

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>> rref(P)

ans =

1.0000    0    0    0    0.5000 -0.5000    0    0
    0    1.0000    0    0    0    0.2500 -0.2500    0
    0    0    1.0000    0    0    0    0.1667 -0.1667
    0    0    0    1.0000    0.1250    0    0    0.1250
```

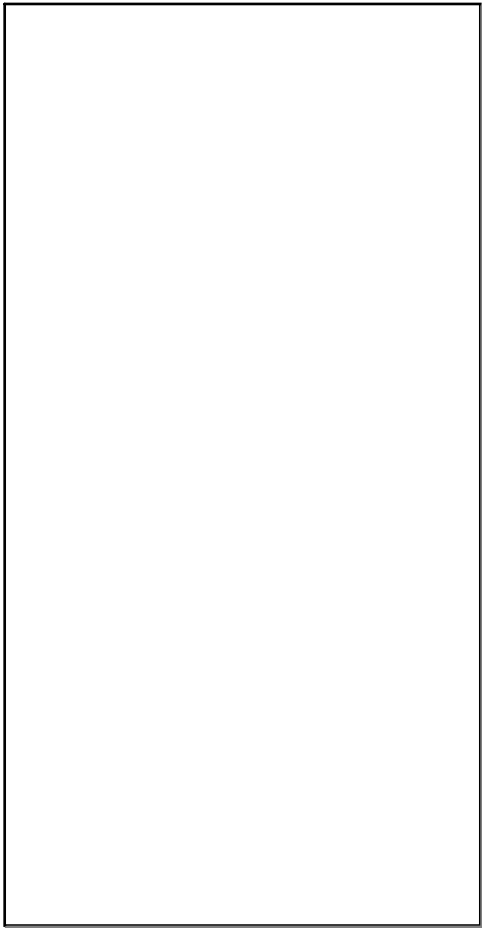
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>> B=ans([1,2,3,4],[5,6,7,8])
```

```
B =

0.5000 -0.5000    0    0
    0    0.2500 -0.2500    0
    0    0    0.1667 -0.1667
0.1250    0    0    0.1250
```

```
>>
```

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