

Linear Algebra  
10/3/2012

Topic: Inverses of matrices

Idea 1: The inverse of an invertible matrix,  $A$ , is the matrix of the inverse of  $T_A$ .

Idea 2: The inverse of an invertible matrix is the product of the elementary matrices used in its reduced row reduction.

Some simple observations:

$$T_A(T_B(\mathbf{v})) = T_{AB}\mathbf{v}, \text{ because } A(B(\mathbf{v})) = (AB)\mathbf{v}.$$

So  $A$  has an inverse  $B$  ( $AB=I=BA$ ) if and only if  $T_A$  is bijective and has inverse  $T_B$ .

So if  $A$  is invertible,  $\text{rref}(A)=I$ .

Since  $\text{rref}(A)=E_nE_{n-1}\dots E_1A$ , if  $A$  is invertible  $E_nE_{n-1}\dots E_1$  is the inverse of  $A$ .

Oct 3-9:07 AM

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Notation: If  $A$  is invertible then its inverse is labelled  $A^{-1}$ .  
So from the previous slide we could write  $E_n E_{n-1} \dots E_1 = A^{-1}$ .  
Recall  $B[A|C] = [BA|BC]$ .  
Notice: if  $A$  is invertible then  $\text{rref}([A|I]) = [I|A^{-1}]$ .

Question: Is this matrix invertible and if so what is its inverse.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

Oct 3-9:25 AM