Linear Algebra
10/25/2012
Topic: More computing of determinants

Idea: the two ways to compute a determinant can be combined to make life easier.

First way: Cofactor expansion

Second way: Forward elimination with no scaling.

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Cofactor expansion example:

1st ROW
4th COLUMN

$$\begin{vmatrix} 1 & 0 & 3 & 2 \\ 5 & 4 & 6 & 0 \\ -1 & -1 & 2 & 0 \\ 3 & -4 & 0 & 0 \end{vmatrix} =$$

1st ROW
3rd COLUMN

2nd ROW
3rd COLUMN

$$(-1)^{1+4} \cdot 2 \cdot \begin{vmatrix} 5 & 4 & 6 \\ -1 & -1 & 2 \\ 3 & -4 & 0 \end{vmatrix} =$$

$$(-1)^{1+4} \cdot 2 \cdot \left((-1)^{1+3} \cdot 6 \cdot \begin{vmatrix} -1 & -1 \\ 3 & -4 \end{vmatrix} + (-1)^{2+3} \cdot 2 \cdot \begin{vmatrix} 5 & 4 \\ 3 & -4 \end{vmatrix} \right)$$

$$= (-2) \cdot \left(6\left((-1)\cdot(-4) - (-1)(3)\right) + (-2)\left(5\cdot(-4) - 4\cdot 3\right) \right)$$

$$= (-2) \cdot \left(6(4+3) + (-2)(-20-12) \right)$$

$$= (-2) \cdot \left(6 \cdot 7 + 2 \cdot (32) \right) = (-2)(42 + 64$$

$$= -212$$

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Forward elimination example:

$$\begin{vmatrix} -1 & -2 & 0 & 3 \\ -3 & 2 & 1 & 4 \\ 5 & 3 & 1 & -5 \\ -4 & 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 5 & 3 & 1 & -5 \\ -4 & 1 & -2 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & -7 & 1 & 10 \\ -4 & 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & -7 & 1 & 10 \\ 0 & 9 & -2 & -9 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & 0 & \frac{15}{8} & \frac{75}{8} \\ 0 & 9 & -2 & -9 \end{vmatrix} = \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & 0 & \frac{15}{8} & \frac{45}{8} \\ 0 & 0 & -\frac{25}{8} & -\frac{27}{8} \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & 0 & \frac{15}{8} & \frac{45}{8} \\ 0 & 0 & 0 & \frac{18}{8} \end{vmatrix} = \begin{vmatrix} -1 & -2 & 0 & 3 \\ 0 & 8 & 1 & -5 \\ 0 & 0 & \frac{15}{8} & \frac{45}{8} \\ 0 & 0 & 0 & 6 \end{vmatrix}$$

$$= -90$$

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A combined approach example:

$$\begin{vmatrix} -1 & 1 & 0 & -2 \\ 0 & 3 & -2 & 1 \\ 4 & -3 & -5 & 0 \\ 4 & -4 & 1 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 0 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 1 & -5 & -8 \\ 4 & -4 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 & 0 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 1 & -5 & -8 \\ 0 & 0 & 1 & -6 \end{vmatrix} = (-1)^{1+1} \cdot (-1) \cdot \begin{vmatrix} 3 & -2 & 1 \\ 1 & -5 & -8 \\ 0 & 1 & -6 \end{vmatrix}$$

$$= (-1) \cdot (-1)^2 (-1) \begin{vmatrix} 1 & -5 & -8 \\ 3 & -2 & 1 \\ 0 & 1 & -6 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 1 & -5 & -8 \\ 0 & 13 & 25 \\ 0 & 1 & -6 \end{vmatrix} = 1 \cdot \begin{vmatrix} 13 & 25 \\ 1 & -6 \end{vmatrix}$$

$$= 13 \cdot (-6) - 25 \cdot 1 = -78 - 25 = -103$$

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ANOTHER METHOD 3x3 ONLY

$$\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & -1 \\ 3 & -2 & -1 \end{vmatrix} = 0 + 2\cdot(-1)\cdot 3 + 3\cdot(-1)\cdot(-2) - 0 - 2(-1)(-1) - (1)(-2)(-1)$$

$$= 0 + (-6) + 6 - 0 - 2 - 2$$

$$= -4$$

$$\left[\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ -1 & 0 & -1 & -1 & 0 \\ 3 & -2 & -1 & 3 & -2 \end{array}\right]$$

$$- \quad - \quad - \quad + \quad + \quad +$$

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REMEMBER!!!

A IS INVERTIBLE
IF AND ONLY IF

$|A| \neq 0$

AND

$|A^{-1}| = |A|^{-1}$ IF IT IS.

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