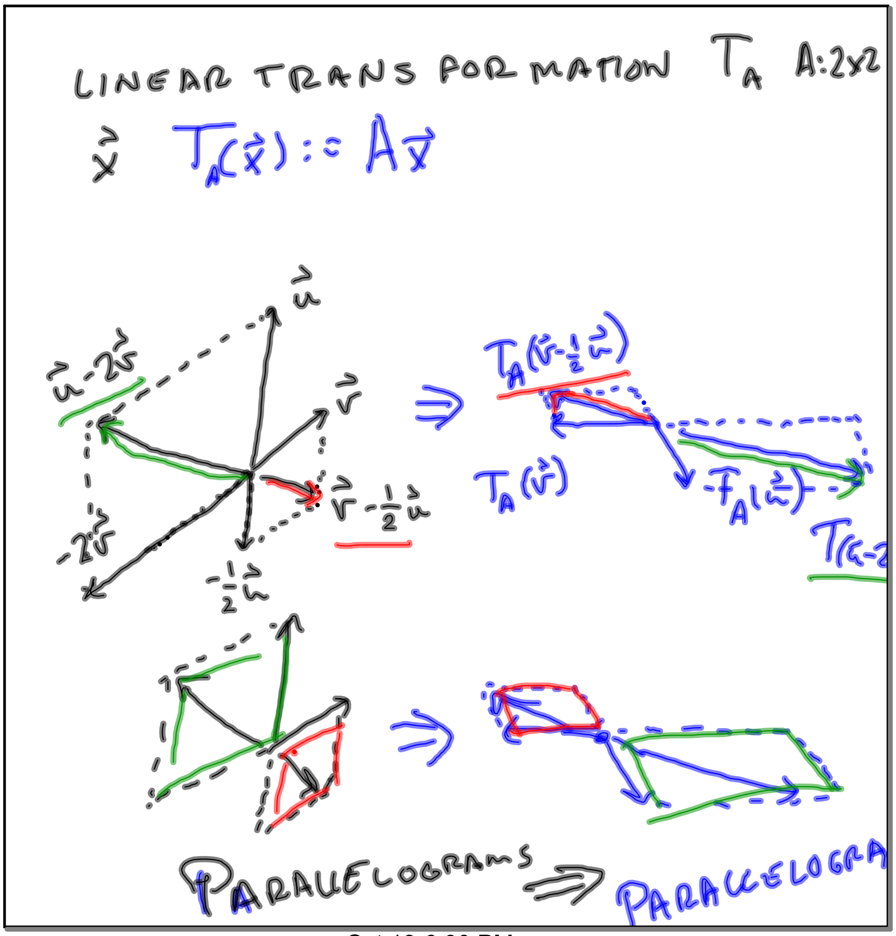
Linear Algebra
10/18/2012

Topic: Determinants

Idea: Determinant of a square matrix is the "change in volumne" times "change in orientation" of its linear transformation.

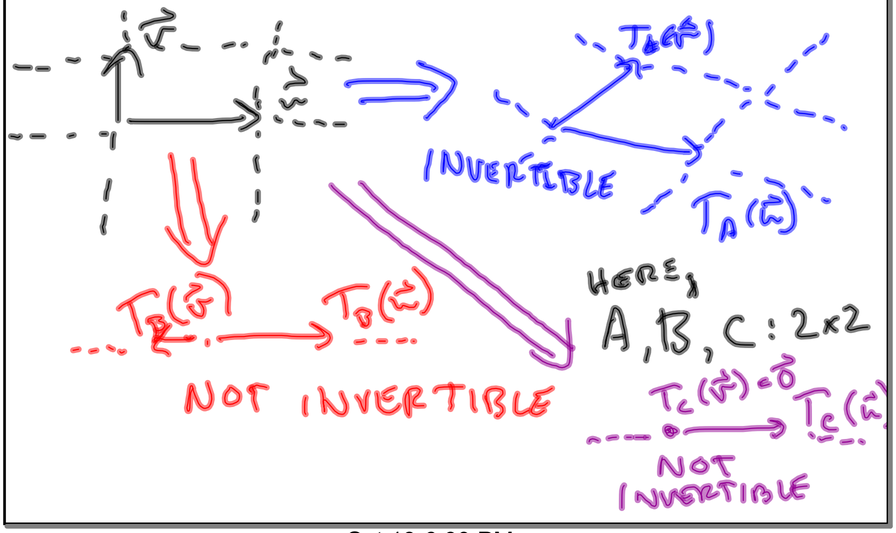
Oct 18-6:10 PM


LINEAR TRANSFORMATION T_A A:2x2
$\vec{x}$ $T_A(\vec{x}) := A\vec{x}$
$\vec{u}$
$\vec{v}$
$\vec{u}-2\vec{v}$
$-2\vec{v}$
$\vec{v}-\frac{1}{2}\vec{u}$
$-\frac{1}{2}\vec{u}$
$T_A(\vec{v}-\frac{1}{2}\vec{u})$
$T_A(\vec{v})$
$-T_A(\vec{u})$
PARALLELOGRAMS ⇒ PARALLELOGRA

Oct 18-6:30 PM

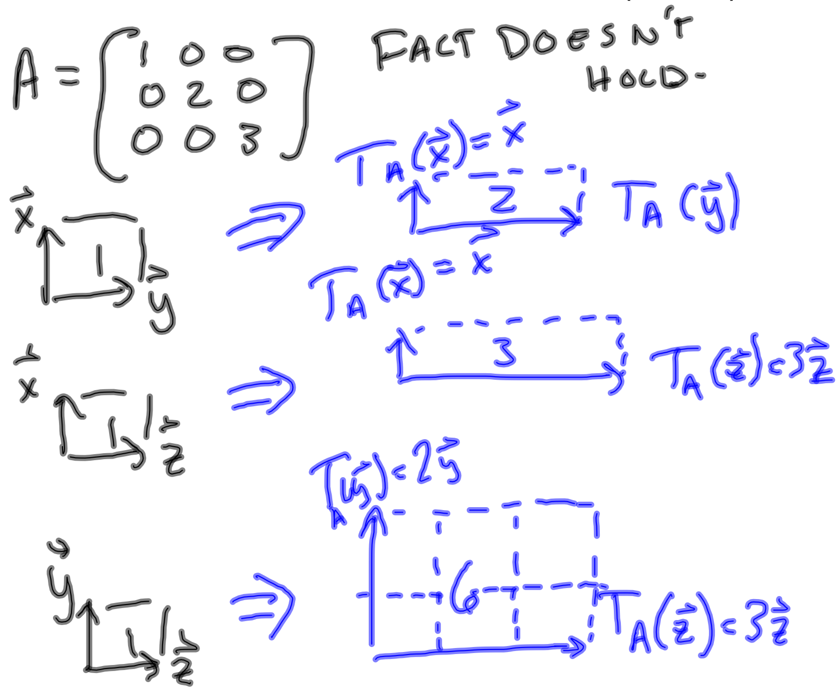
Fact: The scaling of the area of parallelograms is constant for a given matrix A:2x2.
Consequently the scaling of any area is constant for that matrix.

Recall that a square matrix is invertible if and only if it's linear transformation is 1-to-1.
So for a square matrix to be invertible its scaling factor must be non-zero.



Oct 18-6:39 PM

How do we extend this idea to three dimensions (or more)?

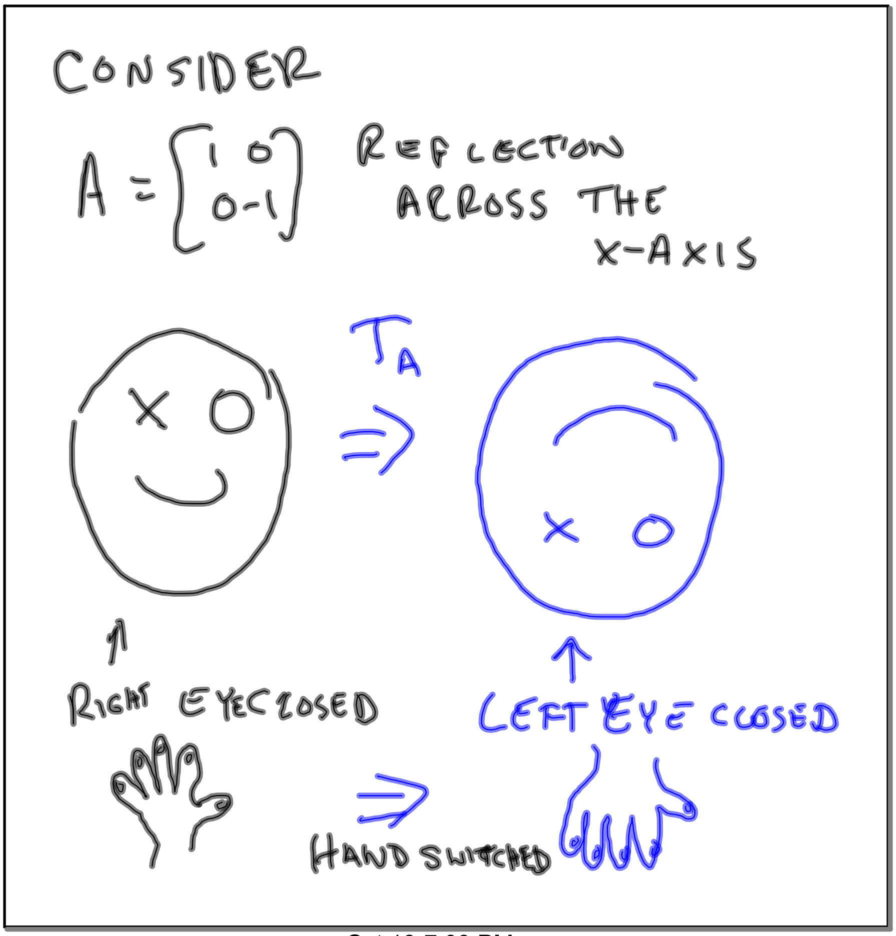


Oct 18-6:53 PM

Fact: The scaling of the (hyper) volume of a (hyper) parallelpiped (page 181) is constant for a given matrix A, square.
Consequently the scaling of any (hyper) volume is constant for that matrix.

Recall that a square matrix is invertible if and only if it's linear transformation is 1-to-1.
So for a square matrix to be invertible its scaling factor must be non-zero.

Oct 18-7:05 PM

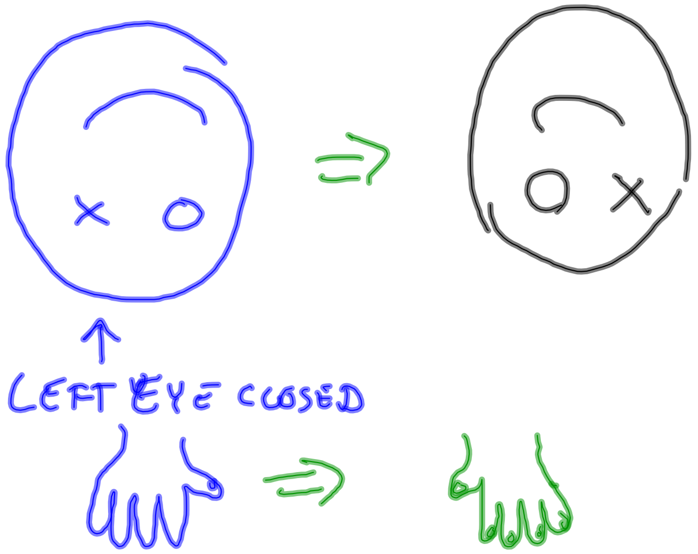

CONSIDER
A = [1 0; 0 -1]
REFLECTION ACROSS THE X-AXIS
T_A
RIGHT EYE CLOSED
LEFT EYE CLOSED
HAND SWITCHED

Oct 18-7:09 PM

This was an example of orientation reversal. Now observe another orientation reversal.

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

REFLECTION ACROSS THE Y AXIS



Oct 18-7:12 PM

It is called reversal because there are only two possible orientations.

The determinant of a square matrix A, det(A) or |A|, is the scaling factor of A times -1 if the orientation is reversed or 1 if it is not.

Consequences:

$|AB|=|A||B|$

A is invertible if and only if |A| is not zero.

$|A^{-1}|=|A|^{-1}$

I want to calculate the volume of the parallelpiped with vectors **v**,**u**, and **w**. How do I do this?

Answer next time after the test.

Oct 18-7:17 PM