Linear Algebra
10/10/2012
Topic: Subspaces and Bases.

Question: What is the image of T_A?
What is the solution to $T_A(\mathbf{x})=\mathbf{0}$?
How do we parameterize these sets?

Idea: Bases are the lists of parametrizing vectors.

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Definitions

A <u>subspace</u> of $\mathbf{R}^n$ is a set of vectors, S, such that if **v** and **u** are in S and r is in **R**, then so too are

- **v**+**u** (additively closed)
- r**v** (closed under scalars)

Note: Every subspace contains **0**. So the smallest subspace is {**0**}.

Example: Solutions to T(**x**)=**0**, because if T(**v**)=**0** and T(**u**)=**0**, then T(r**v**+s**u**)=rT(**v**)+sT(**u**)=**0**+**0**=**0**. This is called the <u>kernel</u> of T.

Example: The image of T, because if **b** and **c** are in the image, and T(**v**)=**b** and T(**w**)=**c**, then T(r**v**+s**w**)=rT(**v**)+sT(**w**)= r**b**+s**c**.

A <u>basis</u> of a subspace, S, is a set of vectors which is linearly independent and spans all of S.

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Note: A basis, B, of a subspace, S has the property:
Any vector in S is representable by a unique linear combination of vectors in B. That is, the weights of such a linear combination are determined by the vector in S.

Two facts:

- The kernel of T_{-A} is the null space of A, that is the solution set to the homogeneous system A**x=0**.
- The image of T_A is the <u>column space</u> of A, that is the set of linear combinations of the columns of A.

EXAMPLE OF NULL SPACE (REVIEW SECT. 1.5)

$$A = \begin{bmatrix} 2 & 3 & 1 & 2 & 1 \\ -2 & 1 & 0 & 1 & -1 \\ 0 & 4 & 1 & 3 & 0 \\ 0 & 4 & 1 & 3 & 1 \end{bmatrix}$$

SOLVE $A\vec{x} = \vec{0}$

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 & -7 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NON-PIVOT COLUMNS

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -7 \\ 3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 5 \end{matrix} \mapsto \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -3 \\ 0 \\ \boxed{1} \\ 0 \end{bmatrix} \right\}$$

BASIS FOR NULL(A)

PIVOT COLUMNS

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