Linear Algerba

Lesson 8
10/1/2012

Topic:
Homogeneous Systems of Linear Equations,
Parametric Vector Form, and
Invertibility of Linear Transformations.

Thought: Any 'functional inverse' of a linear transformation is also a linear transformation.

Oct 1-8:30 AM

Recall Functions:
A function from set D to set C is a relation (a set of pairs the first element from D the second from C) such that each element in D is in exactly one pair.

D is called the domain of such a function. It is the set of inputs.

C is called the codomain (or the target or range). It is a set of possible outputs.

In other words any input of a function has exactly one output.

The image of a function is the set of outputs with a corresponding input.

A function is injective (or one-to-one) if any output has at most one input. ('injection')

A function is surjective (or onto) if the image equals the codomain. ('surjection')

A function is bijective if it is both injective and surjective. ('bijection')

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More on Functions

Given a function, f and an input, x, f(x) is the output corresponding to x.

If f has domain D and codomain C we write f:D->C.

A function, f:D->C is left invertible if there is a function g:C->D such that g(f(x))=x, for all x in D. g is the left inverse of f.

A function, f:D->C is right invertible if there is a function g:C->D such that f(g(y))=y, for all y in C. g is the right inverse of f.

A function, f: D->C is invertible if there is a function g:C->D which is both a left and a right inverse. We say g is the inverse of f.

Theorem: A function f is invertible if and only if it is bijective.
Moreover a function is left invertible if and only if it is injective and
a function is right invertible if and only if it is surjective.

Idea: A linear transformation is (left/right) invertible if is so as a function, that is any (left/right) inverse of a linear transformations is also a linear transformation.

Oct 1-9:02 AM

Homogeneous systems of linear equations:

A homogeneous system of linear equations is one that is equivalent to the matrix equation A**x**=**0**.
Note that if A**v**=**0** then so does rA**v**=A(r**v**)=**0**. (This is why 'homogeneous' is used here.)
Note also that if A**u**=**0** is also true then so is A**v**+A**u**=A(**v**+**u**)=**0**.

Notice that a homogeneous system of linear equations always has the solution **x**=**0**. This is called the trivial solution. Other solutions are called non-trivial.

Theorem: the map T_A (**x**|->A**x**) is injective if and only if the equation A**x**=**0** has no non-trivial solutions, that is it has only the trivial solution.

Proof: If A**v**=**b** and A**u**=**0**, then A(**v**+**u**)=**b**, so if **u** is not zero T_A is not injective. Moreover if A**v**=**b** and A**u**=**b**, then A(**v**-**u**)=**0** so if **v** doesn't equal **u** there is a non-trivial solution to A**x**=**0**.
See the illustration on page 45.

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With injective linear transformations characterized, we want a characterization of surjective linear transformations. That is for which A does A**x**=**b** have a solution for any **b**?

See Theorem 4 on page 37.

Answer: T_A is injective if and only if A has a pivot position in every row.

By the boxed statement on page 43, T_A is surjective if and only if A has a pivot position in every column.

So T_A is bijective if and only if A has a pivot position in every column and every row. In particular A is square.

We want to see that every left or right inverse of T_A as a function is also a linear transformation. Then we can characterize when it is invertible.

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