Find the derivative of each of the following functions using the differentiation rules.

Derivative of a Constant Function: $\frac{d}{dx}(c) = 0$

1a) $f(x) = 60$

$f'(x) = 0$

1b) $y = \frac{8800^{718}}{369} + \frac{40\pi}{47} - 62768.32$

$y' = 0$

The Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ where *n* is any real number

2a) $y = x^5$

$\frac{dy}{dx} = 5x^{5-1} = 5x^4$

2b) $g(x) = x^{-6}$

$g'(x) = -6x^{-6-1}$

$= -6x^{-7}$

The Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

3a) $y = -4x^1$

$\frac{dy}{dx} = -4\frac{d}{dx}(x^1)$

$= -4 \cdot (1x^{1-1})$

$= -4(x^0) = -4$

$\frac{dy}{dx}$ = derivative of y

$\frac{d}{dx}(\ )$ = take the derivative of what's inside

3b) $p(x) = 3x^7$

$p'(x) = 3\frac{d}{dx}(x^7)$

$= 3 \cdot (7x^6)$

$p'(x) = 21x^6$

The Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

The Difference Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

$(\ )'$ = take derivative of function inside parentheses

4a) $y = 2x^{\frac{1}{2}} + 4x^{\frac{2}{3}}$

$y' = (2x^{\frac{1}{2}})' + (4x^{\frac{2}{3}})'$

$y' = 2(x^{\frac{1}{2}})' + 4(x^{\frac{2}{3}})'$

$y' = 2 \cdot \frac{1}{2}x^{\frac{1}{2}-1} + 4 \cdot \frac{2}{3}x^{\frac{2}{3}-1}$

$y' = x^{\frac{1}{2}} + \frac{8}{3}x^{-\frac{1}{3}}$

4b) $t(x) = 2x^{-5} + 4x + 1$

$t'(x) = 2(x^{-5})' + 4(x)' + 1(1)'$

$t'(x) = 2 \cdot -5x^{-5-1} + 4 \cdot 1x^{1-1} + 1(0)$

$t'(x) = -10x^{-6} + 4x^0$

$t'(x) = -10x^{-6} + 4$

4c) $y = -2x^{-4} - 5x^2 - 7x$

$\frac{dy}{dx} = -2 \cdot -4x^{-4-1} - 5 \cdot 2x^{2-1} - 7 \cdot 1x^{1-1}$

$\frac{dy}{dx} = 8x^{-5} - 10x^1 - 7x^0$

$\frac{dy}{dx} = 8x^{-5} - 10x - 7$

4d) $h(x) = 4x^{-7} - 3x^{-1}$

$h'(x) = 4 \cdot -7x^{-7-1} - 3 \cdot -1x^{-1-1}$

$h'(x) = -28x^{-8} + 3x^{-2}$

Review Properties of Exponents		Examples
Product Rule	$x^m \cdot x^n = x^{m+n}$	$y^8 \cdot y^3 = y^{8+3} = y^{11}$
Quotient Rule	$\frac{x^m}{x^n} = x^{m-n}$ where $(x \neq 0)$	$\frac{a^7}{a} = a^{7-1} = a^6$
Zero Exponent	$x^0 = 1$ where $(x \neq 0)$	$w^0 = 1$
Power Rule	$(x^m)^n = x^{m \cdot n}$	$(b^6)^2 = b^{6 \cdot 2} = b^{12}$
Power of a Product	$(x \cdot y)^n = x^n y^n$	$(r^4 t)^3 = (r^4)^3 \cdot t^3 = r^{12} t^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ where $(y \neq 0)$	$\left(\frac{p^9}{q^2}\right)^5 = \frac{p^{9 \cdot 5}}{q^{2 \cdot 5}} = \frac{p^{45}}{q^{10}}$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$ where $(x \neq 0)$	$h^{-3} = \frac{1}{h^3}$
Rational Exponents	$a^{\frac{1}{n}} = \sqrt[n]{a}$ where $\sqrt[n]{a}$ is defined on $\mathbb{R}$	$(64)^{\frac{1}{3}} = \sqrt[3]{64} = \sqrt[3]{4^3} = 4$
Rational Exponents	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where m and n are positive integers and $\sqrt[n]{a}$ is defined on $\mathbb{R}$	$(32)^{\frac{4}{5}} =$ $\sqrt[5]{32^4} = \sqrt[5]{(2^5)^4} = \sqrt[5]{2^{20}} = \sqrt[5]{(2^4)^5} = 2^4 = 16$; $\left(\sqrt[5]{32}\right)^4 = 2^4 = 16$

5. For each exercise below,
   i. Simplify and rewrite each term as x^n with exponent in the numerator.
   ii. Find the derivative.

Function	Rewriting the exponents	Derivative
$y = \frac{1}{x^2} + \frac{1}{x}$	$y = x^{-2} + x^{-1}$	$y' = -2x^{-3} - x^{-2}$

Function	Rewriting the exponents	Derivative
$y = \frac{1}{\sqrt{x}} - \sqrt{x}$	$y = x^{-\frac{1}{2}} - x^{\frac{1}{2}}$	$y' = -\frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$
$y = \frac{1}{\sqrt[3]{x}} + \left(\sqrt{x}\right)^3$	$y = \frac{1}{x^{\frac{1}{3}}} + \left(x^{\frac{1}{2}}\right)^3$ $y = x^{-\frac{1}{3}} + x^{\frac{3}{2}}$	$y' = -\frac{1}{3}x^{-\frac{4}{3}} + \frac{3}{2}x^{\frac{1}{2}}$
$y = \left(x^{\frac{2}{3}}\right)^{\frac{5}{8}}$	$y = x^{\frac{\cancel{2}^{1}}{3}\cdot\frac{5}{\cancel{8}_{4}}} = x^{\frac{5}{12}}$	$\frac{dy}{dx} = \frac{5}{12}x^{-\frac{7}{12}}$
$y = \frac{x^3 + 3x^2 + 6 + 7x^{-3}}{x^2}$	$y = \frac{x^3}{x^2} + \frac{3x^2}{x^2} + \frac{6}{x^2} + \frac{7x^{-3}}{x^2}$ $y = x + 3 + 6x^{-2} + 7x^{-5}$	$y' = (x^1)' + (3)' + (6x^{-2})' + (7x^{-5})'$ $y' = 1x^{1-1} + 0 + 6\cdot -2x^{-2-1} + 7\cdot -5x^{-5-1}$ $y' = 1 + 0 - 12x^{-3} - 35x^{-6}$ $y' = 1 - 12x^{-3} - 35x^{-6}$
$y = \frac{2x^3 - x^2 + 3x - 5}{\sqrt{x}}$	$y = \frac{2x^3}{x^{\frac{1}{2}}} - \frac{x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} - \frac{5}{x^{\frac{1}{2}}}$ $y = 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 5x^{-\frac{1}{2}}$	$\frac{dy}{dx} = 2\cdot\frac{5}{2}x^{\frac{5}{2}-1} - \frac{3}{2}x^{\frac{3}{2}-1} + 3\cdot\frac{1}{2}x^{\frac{1}{2}-1} - 5\cdot\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$ $\frac{dy}{dx} = 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + \frac{5}{2}x^{-\frac{3}{2}}$

The Product Rule: $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)]\cdot g(x) + \frac{d}{dx}[g(x)]\cdot f(x)$

or

$(fg)' = f'g + g'f$ $\quad f'g + fg'$

6a) $y = x^4(2x+3)$

$f = x^4$	$g = 2x+3$
$f' = 4x^3$	$g' = 2$

$\frac{dy}{dx} = f'g + fg'$

$= 4x^3(2x+3) + x^4(2)$

$= 8x^4 + 12x^3 + 2x^4$

$= 10x^4 + 12x^3$

6b) $g(x) = (3x-7)(x^2+6x)$

$u = 3x-7$	$v = x^2+6x$
$u' = 3$	$v' = 2x+6$

$g'(x) = u'v + uv'$

$= 3(x^2+6x) + (3x-7)(2x+6)$

$= 3x^2 + 18x + 6x^2 + 18x - 14x - 42$

$g'(x) = 9x^2 + 22x - 42$

The Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]\cdot g(x) - \frac{d}{dx}[g(x)]\cdot f(x)}{[g(x)]^2}$

or

$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2} = \frac{f'g - fg'}{g^2}$

7a) $y = \frac{x}{3x+1}$ $\begin{matrix}\leftarrow f \\ \leftarrow g\end{matrix}$

$f = x$	$g = 3x+1$
$f' = 1$	$g' = 3$

$\frac{dy}{dx} = \frac{f'g - fg'}{g^2}$

$= \frac{1\cdot(3x+1) - 3x}{(3x+1)^2}$

$\frac{dy}{dx} = \frac{1}{(3x+1)^2}$

7b) $q(x) = \frac{9x^2}{3x^2 - 2x}$

$f = 9x^2$	$g = 3x^2 - 2x$
$f' = 18x$	$g' = 6x - 2$

$q'(x) = \frac{f'g - fg'}{g^2}$

$q'(x) = \frac{18x(3x^2-2x) - 9x^2(6x-2)}{(3x^2-2x)^2}$

$q'(x) = \frac{54x^3 - 36x^2 - 54x^3 + 18x^2}{(3x^2-2x)^2}$

$q'(x) = \frac{-18x^2}{(3x^2-x)^2}$

$q'(x) = -\frac{18x^2}{x^2(3x-1)^2}$

$q'(x) = -\frac{18}{(3x-1)^2}$

The Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ $\quad f'(u) \cdot u'$

Derivative of the composite function $f(g(x))$

Derivative of the outside function f

Derivative of the inside function g

If y is a function of u, and u is a function of x, then the chain rule is $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

8. Find the derivative using the Chain Rule

a) $y = (x^2+3)^8$

$x \to u = x^2+3 \to f(u) = u^8$

$y' = f'(u) \cdot u'$

$= 8u^7 \cdot u'$

$= 8(x^2+3)^7 \cdot (x^2+3)'$

$= 8(x^2+3)^7 \cdot 2x$

$y' = 16x(x+3)^7$

b) $s(x) = 2(5x-500)^{1000}$

x

$\to u = 5x-500$

$\to f(u) = u^{1000}$

$s'(x) = 2 \cdot \frac{dy}{dx}(5x-500)^{1000}$

$s'(x) = 2 \cdot f'(u) \cdot u'$

$s'(x) = 2 \cdot 1000u^{999} \cdot u'$

$s'(x) = 2000 \, (5x-500)^{999} \quad (5x-500)'$

$s'(x) = 2000 \, (5x-500)^{999} \cdot 5$

$s'(x) = 10\,000 \, (5x-500)^{999}$

c) $y = \sqrt{3x+1}$

x

$\to u = 3x+1$

$\to f(u) = \sqrt{u}$

$= u^{\frac{1}{2}}$

$y' = f'(u) \cdot u'$

$= \frac{1}{2} u^{-\frac{1}{2}} \cdot u'$

$= \frac{1}{2}(3x+1)^{-\frac{1}{2}} \cdot (3x+1)'$

$= \frac{1}{2}(3x+1)^{-\frac{1}{2}} \cdot 3$

$= \frac{3}{2}(3x+1)^{-\frac{1}{2}}$

$y' = \frac{3}{2\sqrt{3x+1}}$

d) $y = \frac{8}{\sqrt{5x^2+1}}$

x

$\to u = 5x^2+1$

$\to f(u) = \frac{1}{\sqrt{u}}$

$= \frac{1}{u^{\frac{1}{2}}}$

$= u^{-\frac{1}{2}}$

$\frac{dy}{dx} = 8 \frac{d}{dx}\left(\frac{1}{\sqrt{5x^2+1}}\right)$

$\frac{dy}{dx} = 8 \cdot f'(u) \cdot u'$

$\frac{dy}{dx} = 8 \cdot -\frac{1}{2} u^{-\frac{3}{2}} \cdot u'$

$\frac{dy}{dx} = 8 \cdot \left(-\frac{1}{2}(5x^2+1)^{-\frac{3}{2}}\right) \cdot 10x$

$\frac{dy}{dx} = -40x(5x^2+1)^{-\frac{3}{2}}$

9. Find the derivative using combination rules

a) $y = -4x^2(2x^3 - 14)^4$

$f = -4x^2$ | $g = (2x^3-14)^4$

$f' = -8x$ | $g' = 4(2x^3-14)^3 \cdot 6x^2$

$= 24x^2(2x^3-14)^3$

$y' = f'g + fg'$

$y' = -8x(2x^3-14)^4 + (-4x^2)24x^2(2x^3-14)^3$

$u = 2x^3-14 \rightarrow f'(u)\cdot u' = 4u^3 \cdot u'$

$f(u) = u^4$

$= 4(2x^3-14)^3\cdot(2x^3-14)'$

$= 4(2x^3-14)^3\cdot(6x^2)$

$= 24x^2(2x^3-14)^3$

b) $y = x^3 \cdot \sqrt{x^2+3}$

c) $y = \frac{\sqrt{x^2+3}}{x^3}$

$f = (x^2+3)^{\frac{1}{2}}$ | $g = x^3$

$f' = x(x^2+3)^{-\frac{1}{2}}$ | $g' = 3x^2$

$y' = \frac{f'g - fg'}{g^2}$

$y' = \frac{x^4(x^2+3)^{-\frac{1}{2}} - 3x^2(x^2+3)^{\frac{1}{2}}}{x^6}$

$u = x^2+3$

$h(u) = u^{\frac{1}{2}} \rightarrow f' = h'(u)\cdot u'$

$= \frac{1}{2}(x^2+3)^{-\frac{1}{2}}\cdot(x^2+3)'$

$= \frac{1}{\cancel{2}}(x^2+3)^{-\frac{1}{2}}(\cancel{2}x)$

$= x(x^2+3)^{-\frac{1}{2}}$