Method 3: Rationalizing Technique – If the function has a radical expression in the numerator, rationalize the numerator by multiplying in the numerator and denominator by the "conjugate of the numerator."

Exercise 3: Find the limit.

a) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

$$\frac{\sqrt{x+3}-\sqrt{3}}{x} = \frac{\sqrt{x+3}-\sqrt{3}}{x} \cdot \frac{\sqrt{x+3}+\sqrt{3}}{\sqrt{x+3}+\sqrt{3}}$$

(multiply by conjugate of numerator)

$$a^2 - b^2 = (a+b)(a-b)$$

conjugates of each other

$$= \frac{(\sqrt{x+3})^2 - (\sqrt{3})^2}{x(\sqrt{x+3}+\sqrt{3})}$$

$$= \frac{x+\cancel{3}-\cancel{3}}{x(\sqrt{x+3}+\sqrt{3})}$$

$$= \frac{\cancel{x}}{\cancel{x}(\sqrt{x+3}+\sqrt{3})}$$

$$= \frac{1}{\sqrt{x+3}+\sqrt{3}}$$

$$\to \lim_{x\to 0} \frac{1}{\sqrt{x+3}+\sqrt{3}}$$

$$= \frac{1}{\sqrt{0+3}+\sqrt{3}}$$

$$= \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

b) $\lim_{x\to 4} \left(\frac{\sqrt{x+5}-3}{x-4}\right)\left(\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}\right)$

$$= \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$$

$$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x+5}+3)}$$

$$= \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3}$$

$$= \frac{1}{\sqrt{(4)+5}+3} = \boxed{\frac{1}{6}}$$

Method 4: The LCD Technique - Combining fractions in the numerator using the Least Common Denominator (LCD)

Exercise 4: Find the limit.

a) $\lim\limits_{x\to0}\dfrac{\frac{1}{x+5}-\frac{1}{5}}{x} = \lim\limits_{x\to0}\dfrac{\frac{1}{x+5}-\frac{1}{5}}{x}\left(\dfrac{5(x+5)}{5(x+5)}\right)$

LCD: $5(x+5)$

$$= \lim_{x\to0}\frac{5-(x+5)}{5x(x+5)}$$

$$= \lim_{x\to0}\frac{\cancel{5}-x-\cancel{5}}{5x(x+5)}$$

$$= \lim_{x\to0}\frac{-\cancel{x}}{5\cancel{x}(x+5)}$$

$$= \lim_{x\to0}-\frac{1}{5(x+5)}$$

$$= -\frac{1}{5((0)+5)}$$

$$= -\frac{1}{25}$$

b) $\lim\limits_{x\to2}\dfrac{\frac{1}{x-4}+\frac{1}{2}}{x-2} = \lim\limits_{x\to2}\dfrac{\frac{1}{x-4}+\frac{1}{2}}{x-2}\left(\dfrac{2(x-4)}{2(x-4)}\right)$

$$\frac{2(x-4)}{x-4}+\frac{2(x-4)}{2}$$

LCD: $2(x-4)$

$$= \lim_{x\to2}\frac{2+(x-4)}{2(x-2)(x-4)}$$

$$= \lim_{x\to2}\frac{\cancel{(x-2)}\,1}{2\cancel{(x-2)}(x-4)}$$

$$= \frac{1}{2((2)-4)} = \boxed{-\frac{1}{4}}$$

SECTION 2 SUPPLEMENTARY EXERCISES

Find the limit.

a) $\lim_{x \to -4} (-x^3 + 5x^2)$

b) $\lim_{x \to 6} \sqrt{2x+4}$

c) $\lim_{x \to -1} \frac{x^2 + 6x - 7}{x + 2}$

d) $\lim_{x \to 0} \frac{x^2 + 7x}{x}$

e) $\lim_{x \to 5} \frac{x^2 - 3x - 10}{x^2 - 8x + 15}$

f) $\lim_{x \to 0} \frac{\frac{1}{x-6} + \frac{1}{6}}{3x}$

g) $\lim_{x \to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$

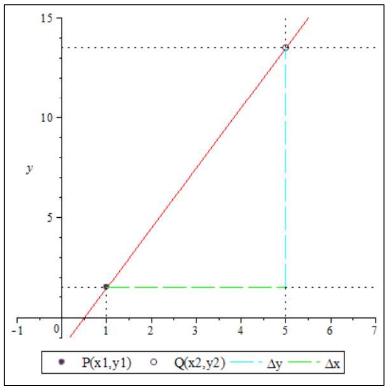
h) $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1}}{x}$

Section 3: The Derivative of a Function

Calculus is primarily the study of change. The basic focus on calculus is divided in to two categories, Differential Calculus and Integral Calculus. In this section we will introduce differential calculus, the study of rate at which something changes.
Consider the example at which x is to be the independent variable and y the dependent variable. If there is any change Δx in the value of x, this will result in a change Δy in the value of y. The resulting change in y for each unit of change in x remains constant and is called the slope of the line.

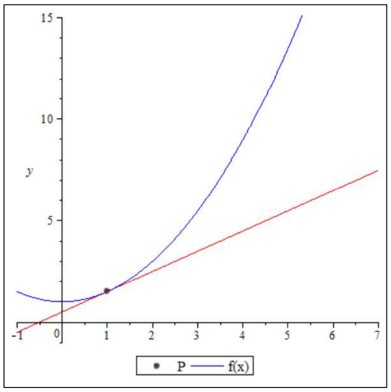
The slope of a straight line is represented as:

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in the } y \text{ coordinate}}{\text{Change in the } x \text{ coordinate}}$$



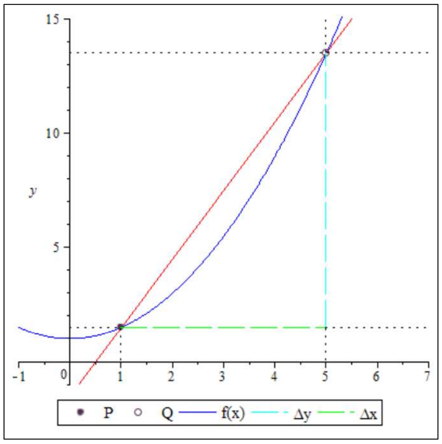
TANGENT LINES

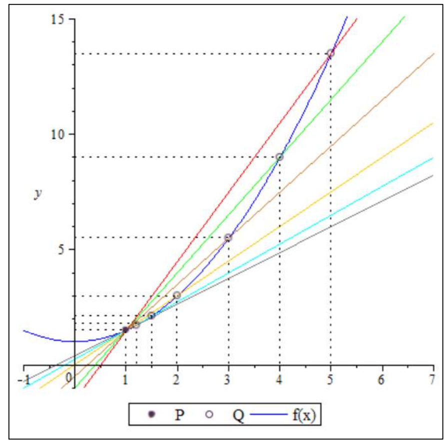
Calculus is concerned with the rate of change that is not constant. Therefore, it is not possible to determine a slope that satisfies every point of the curve. The question that calculus presents is: "What is the rate of change at the point P?" And we can find the slope of the tangent line to the curve at point P by the method of differentiation. A tangent line at a given point to a plane curve is a straight line that touches the curve at that point.



SECANT LINES

Like a tangent line, a secant line is also a straight line; however a secant line passes through two points of a given curve.





Therefore we must consider an infinite sequence of shorter intervals of Δx, resulting in an infinite sequence of slopes. We define the tangent to be the limit of the infinite sequence of slopes. The value of this limit is called the derivative of the given function.

$$\text{The slope of the tangent at P} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Secant line graph
https://www.desmos.com/calculator/pzlb0a2z3v

Go to Desmos link above to see how secant line works. The red dot represents point P, the blue dot represents point Q. Slide the blue dot Q towards the red dot P to see how the secant line PQ becomes the tangent line at P when the distance between PQ approaches zero.

Tangent Line graph
https://www.desmos.com/calculator/dxg5fjxwb7

Go to Desmos link above to see how tangent line changes as it traverses along a function f.

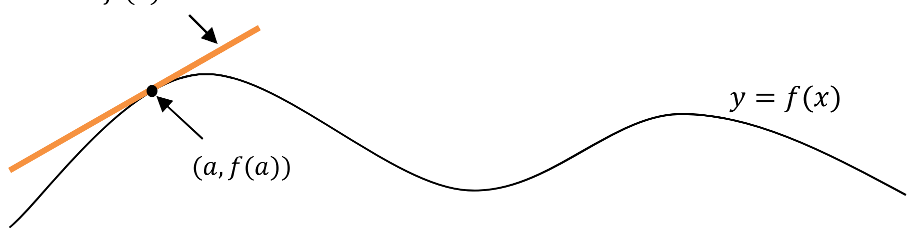
In your own words, what is the tangent line to a function?

____

____

What does derivative of $f(x)$ mean graphically?

It is the slope (m) of the tangent line of the graph $\boldsymbol{y = f(x)}$ at the point $(a, f(a))$



THE DEFINITION OF THE DERIVATIVE

THE DIFFERENCE QUOTIENT

To find the slope of the tangent line to the function $y = f(x)$ at, we must choose a point of tangency, $(x, f(x))$ and a second point $(x + h, f(x + h))$, where $h = \Delta x$.

The slope of the tangent line, or the derivative of a function f is defined as:

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{(x + h) - h} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

derivative function of x. ($f'(x)$, y', $\frac{dy}{dx}$, $\frac{df}{dx}$ ...)

Example 1: Find the derivative of the function $f(x) = 7x + 11$ using the definition of derivative.

Let $f(x) = 7x + 11$
And $f(x + h) = 7(x + h) + 11$

By the definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{7(x + h) + 11 - (7x + 11)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{7x + 7h + 11 - 7x - 11}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{7h}{h} = \lim_{h \to 0}(7) = 7$$

Therefore, the derivative of $f(x)$ is 7.

Example 2: Find the derivative of the function $g(x) = 3x^2 + 6x - 9$ using the definition of derivative.

Let $g(x) = 3x^2 + 6x - 9$
And $g(x+h) = 3(x+h)^2 + 6(x+h) - 9$

By the definition of the derivative:

$$g'(x) = \lim_{h \to 0} \frac{3(x+h)^2 + 6(x+h) - 9 - (3x^2 + 6x - 9)}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 6x + 6h - 9 - 3x^2 - 6x + 9}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 6x + 6h - 9 - 3x^2 - 6x + 9}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 6h}{h}$$

$$g'(x) = \lim_{h \to 0} (6x + 3h + 6)$$

$$g'(x) = 6x + 3(0) + 6$$

$$g'(x) = 6x + 6$$

Therefore the derivative of $g(x)$ is $6x + 6$.

Exercise 1: Find the derivative of the following functions using the definition of derivative.

a) $f(x) = 5x + 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5h}{h}$$

$$= \lim_{h \to 0} 5$$

$$= \boxed{5}$$

$$f(x+h) = 5(x+h) + 2$$
$$= 5x + 5h + 2$$
$$-f(x) = -5x \quad -2$$
$$5h$$

b) $f(x) = 3x^2 - 4x$

$$f(x+h) = 3(x+h)^2 - 4(x+h)$$
$$= 3(x^2+2xh+h^2) - 4x - 4h$$
$$= 3x^2 + 6xh + 3h^2 - 4x - 4h$$
$$-f(x) = -3x^2 \qquad +4x$$
$$\overline{\qquad 6xh + 3h^2 \quad -4h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{6xh+3h^2-4h}{h}$$
$$= \lim_{h\to 0} (6x+3h-4)$$
$$= 6x+3(0)-4$$
$$f'(x) = 6x-4$$

c) $f(x) = \sqrt{x}$

$$f(x+h) = \sqrt{x+h}$$
$$-f(x) = \sqrt{x}$$
$$\overline{\sqrt{x+h} - \sqrt{x}}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$
$$= \lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}\;1}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+(0)}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}} = \boxed{\frac{\sqrt{x}}{2x}}$$

d) $f(x) = \frac{1}{x}$

Example 3. Find the slope of the tangent line to the curve $f(x)=x^2+2$ at the point $(-1,3)$ using the definition of the derivative, and find the equation of the tangent line.

By definition, the slope of the tangent line at any point is given by $f'(x)$.
Therefore $f'(x)$ equals to the following:

$$m = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$m = \lim_{h\to 0}\frac{((x+h)^2+2)-(x^2+2)}{h}$$
$$m = \lim_{h\to 0}\frac{(x^2+2xh+h^2+2)-x^2-2}{h}$$
$$m = \lim_{h\to 0}\frac{2xh+h^2}{h}$$
$$m = \lim_{h\to 0}\frac{h(2x+h)}{h}$$
$$m = \lim_{h\to 0}(2x+h)$$
$$m = 2x+0$$
$$m = 2x$$

Now this is the slope of the tangent at a point $(x, f(x))$ of the graph. Since the line is tangent at $(-1,3)$, we have to evaluate m at $(-1,3)$. Therefore, $m=2(-1)=-2$.
The slope of the tangent at $(-1,3)$ is -2.

Equation of a Line Review

Slope-Intercept Form

The equation of any line with slope m and y-intercept b is given by $y = mx + b$

$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$ $b = \text{y} - \text{intercept}$

Point-Slope Form

The equation of the line through (x_1, y_1) with slope m is given by $y - y_1 = m(x - x_1)$

To find the equation of the tangent line to the curve $f(x)=x^2+2$ use the point-slope formula to find the equation:

$$y-(3)=-2(x-(-1))$$
$$y-3=-2x-2$$
$$y=-2x+1$$

The equation of the tangent line to the curve $f(x)=x^2+2$ at the point $(-1,3)$ is $y=-2x+1$.

$(x, f(x))$

Exercise 2: Find the slope of the tangent line to the curve $f(x) = -3x^2 + x$ at the point $(2, -10)$ using the definition of the derivative, and find the equation of the tangent line.

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-6xh - 3h^2 + h}{h}$$

$$f(x+h) = -3(x+h)^2 + (x+h)$$
$$= -3x^2 - 6xh - 3h^2 + x + h$$
$$-f(x) = +3x^2 \quad -x$$
$$-6xh - 3h^2 \quad + h$$

$$= \lim_{h \to 0} (-6x - 3h + 1)$$
$$= -6x - 3(0) + 1$$
$$= -6x + 1$$

Slope

$$m = f'(2) = -6(2) + 1$$
$$= -11$$

Tangent line (Point slope form $y = m(x - x_1) + y_1$)

$$y = -11(x - (2)) + (-10)$$
$$y = -11x + 22 - 10$$
$$\boxed{y = -11x + 12}$$

Exercise 3: Find the slope of the tangent line to the curve $f(x) = x^3$ at the point $(-2, -8)$ using the definition of the derivative, and find the equation of the tangent line.

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$f(x+h) = (x+h)^3$$
$$= x^3 + 3x^2h + 3xh^2 + h^3$$
$$-f(x) = -x^3$$
$$3x^2h + 3xh^2 + h^3$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2)$$
$$= 3x^2 + 3x(0) + (0)^2$$
$$= 3x^2$$

slope

$$m = f'(-2)$$
$$= 3(-2)^2$$
$$= 12$$

Tangent line: $y = m(x - x_1) + y_1$

$$y = (12)(x - (-2)) + (-8)$$
$$y = 12x + 24 - 8$$
$$\boxed{y = 12x + 16}$$

SECTION 3 SUPPLEMENTARY EXERCISES

1. Use the definition of derivative $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to determine the derivative of each function.

   a) $g(x) = \sqrt{1-5x}$

   b) $h(y) = \dfrac{6y}{y+1}$

   c) $p(x) = \sqrt{x} + x$

   d) $k(t) = \dfrac{1}{t^2}$

2. Find the slope of the tangent line to the curve $f(x) = \dfrac{1}{x-6}$ at the point $(5,-1)$ using the definition of the derivative, and find the equation of the tangent line.

Section 4: Differentiation Rules

The Differentiation Formulas

Derivative of a Constant Function	$\frac{d}{dx}(c) = 0$	
The Power Rule	$\frac{d}{dx}(x^n) = nx^{n-1}$ where *n* is any real number	
The Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$	
The Sum Rule	$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$	
The Difference Rule	$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$	
The Product Rule	$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)]\cdot g(x) + \frac{d}{dx}[g(x)]\cdot f(x)$	Or in prime notation $(fg)' = f'g + g'f$
The Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]\cdot g(x) - \frac{d}{dx}[g(x)]\cdot f(x)}{[g(x)]^2}$	Or in prime notation $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$
The Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)$	Or, if y is a function of u, and u is a function of x, then $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

Find the derivative of each of the following functions using the differentiation rules.

Derivative of a Constant Function: $\frac{d}{dx}(c) = 0$

1a) $f(x) = 60$

$f'(x) = 0$

1b) $y = \frac{8800^{718}}{369} + \frac{40\pi}{47} - 62768.32$

$\frac{dy}{dx} = 0$

* The Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ where *n* is any real number

2a) $y = x^5$

$y' = 5x^{5-1}$

$y' = 5x^4$

2b) $g(x) = x^{-6}$

$g'(x) = -6x^{-6-1}$

$g'(x) = -6x^{-7}$

The Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

3a) $y = -4x$

$\frac{dy}{dx} = -4\frac{d}{dx}(x^1)$

$= -4(1x^{1-1})$

$= -4(x^0) = -4(1)$

$= -4$

3b) $p(x) = 3x^7$

$p'(x) = 3\frac{d}{dx}(x^7)$

$= 3(7x^6)$

$= 21x^6$

The Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

The Difference Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

4a) $y = 2x^{\frac{1}{2}} + 4x^{\frac{2}{3}}$

$y' = \frac{d}{dx}(2x^{\frac{1}{2}}) + \frac{d}{dx}(4x^{\frac{2}{3}})$

$y' = 2\frac{d}{dx}(x^{\frac{1}{2}}) + 4\frac{d}{dx}(x^{\frac{2}{3}})$

$y' = 2(\frac{1}{2}x^{-\frac{1}{2}}) + 4(\frac{2}{3}x^{-\frac{1}{3}})$

$y' = x^{-\frac{1}{2}} + \frac{8}{3}x^{-\frac{1}{3}}$

4b) $t(x) = 2x^{-5} + 4x + 1$

$t'(x) = -10x^{-6} + 4$

4c) $y = -2x^{-4} - 5x^2 - 7x$

$y' = 8x^{-5} - 10x - 7$

4d) $h(x) = 4x^{-7} - 3x^{-1}$

$h'(x) = -28x^{-8} + 3x^{-2}$

Review Properties of Exponents		Examples
Product Rule	$x^m \cdot x^n = x^{m+n}$	$y^8 \cdot y^3 = y^{8+3} = y^{11}$
Quotient Rule	$\frac{x^m}{x^n} = x^{m-n}$ where $(x \neq 0)$	$\frac{a^7}{a} = a^{7-1} = a^6$
Zero Exponent	$x^0 = 1$ where $(x \neq 0)$	$w^0 = 1$
Power Rule	$(x^m)^n = x^{m \cdot n}$	$(b^6)^2 = b^{6\cdot 2} = b^{12}$
Power of a Product	$(x \cdot y)^n = x^n y^n$	$(r^4 t)^3 = (r^4)^3 t^3 = r^{12} t^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ where $(y \neq 0)$	$\left(\frac{p^9}{q^2}\right)^5 = \frac{p^{45}}{q^{10}}$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$ where $(x \neq 0)$	$h^{-3} = \frac{1}{h^3}$
Rational Exponents	$a^{\frac{1}{n}} = \sqrt[n]{a}$ where $\sqrt[n]{a}$ is defined on $\mathbb{R}$	$(64)^{\frac{1}{3}} = \sqrt[3]{64} = 4$
Rational Exponents	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ where m and n are positive integers and $\sqrt[n]{a}$ is defined on $\mathbb{R}$	$(32)^{\frac{4}{5}} = \sqrt[5]{32^4}$ / $\left(\sqrt[5]{32}\right)^4$ $= 16$

5. For each exercise below,
   i. Simplify and rewrite each term as x^n with exponent in the numerator.
   ii. Find the derivative.

Function	Rewriting the exponents	Derivative
$y = \frac{1}{x^2} + \frac{1}{x}$	$y = x^{-2} + x^{-1}$	$y' = -2x^{-3} - x^{-2}$

Function	Rewriting the exponents	Derivative
$y = \frac{1}{\sqrt{x}} - \sqrt{x}$	$y = x^{-\frac{1}{2}} - x^{\frac{1}{2}}$	$y' = -\frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$
$y = \frac{1}{\sqrt[3]{x}} + \left(\sqrt{x}\right)^3$	$y = x^{-\frac{1}{3}} + x^{\frac{3}{2}}$	$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{4}{3}} + \frac{3}{2}x^{\frac{1}{2}}$
$y = \left(x^{\frac{\cancel{2}\,1}{3}}\right)^{\frac{5}{\cancel{8}\,4}}$	$y = x^{\frac{5}{12}}$	$\frac{dy}{dx} = \frac{5}{12}x^{\frac{5}{12}-1}$ $\frac{dy}{dx} = \frac{5}{12}x^{-\frac{7}{12}}$
$y = \frac{x^3 + 3x^2 + 6 + 7x^{-3}}{x^2}$	$y = \frac{x^3}{x^2} + \frac{3x^2}{x^2} + \frac{6}{x^2} + \frac{7x^{-3}}{x^2}$ $y = x + 3 + 6x^{-2} + 7x^{-5}$	$y' = 1 - 12x^{-3} - 35x^{-6}$
$y = \frac{2x^3 - x^2 + 3x - 5}{\sqrt{x}}$	$y = \frac{2x^3}{x^{\frac{1}{2}}} - \frac{x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} - \frac{5}{x^{\frac{1}{2}}}$ $y = 2x^{\frac{5}{2}} - x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 5x^{-\frac{1}{2}}$	$\frac{dy}{dx} = 5x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + \frac{5}{2}x^{-\frac{3}{2}}$

The Product Rule: $\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + \frac{d}{dx}[g(x)] \cdot f(x)$

or

$$(fg)' = f'g + g'f$$

6a) $y = x^4(2x+3)$

Product Rule

$f = 2x+3$	$g = x^4$
$f' = 2$	$g' = 4x^3$

$y' = f'g + fg'$

$y' = 2x^4 + (2x+3) \cdot 4x^3$

$y' = 2x^4 + 4x^3(2x+3)$

$y = 2x^4 + 8x^4 + 12x^3$

$\boxed{y' = 10x^4 + 12x^3}$

6b) $g(x) = (3x-7)(x^2+6x)$

Product Rule

$f = 3x-7$	$g = x^2+6x$
$f' = 3$	$g' = 2x+6$

$g'(x) = (3x-7)(2x+6) + 3(x^2+6x)$

The Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - \frac{d}{dx}[g(x)] \cdot f(x)}{[g(x)]^2}$

or

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

7a) $y = \frac{x}{3x+1}$

7b) $q(x) = \frac{9x^2}{3x^2 - 2x}$