Long Division

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x-1 & x^3 + 0x^2 + 0x - 1 \\
 & +(-x^3 + x^2) \\
\hline
 & \quad + x^2 + 0x \\
 & \quad + (-x^2 + x) \\
\hline
 & \qquad + x - 1 \\
 & \qquad + (-x + 1) \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$\frac{x^3}{x} = x^2$$

$$\rightarrow x^2(x-1) = x^3 - x^2$$

$$\frac{x^2}{x} = x$$

$$\rightarrow x(x-1) = x^2 - x$$

$$\frac{x}{x} = 1$$

$$\rightarrow 1(x-1) = x - 1$$

$$\rightarrow \frac{x^3-1}{x-1} = x^2 + x + 1 + \frac{0}{x-1}$$

$$= x^2 + x + 1$$

Synthetic Division

$x - 1 = 0$

$x = 1$

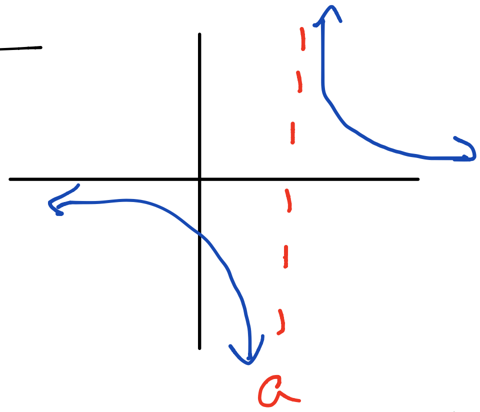
$$
\begin{array}{c|cccc}
 & x^3 & x^2 & x & c \\
1 & 1 & 0 & 0 & -1 \\
 & \downarrow & 1 & 1 & 1 \\
\hline
 & 1 & 1 & 1 & \boxed{0}
\end{array}
\leftarrow \text{Remainder}
$$

$$x^2 \quad x \quad c \rightarrow 1x^2 + 1x + 1$$

$$= x^2 + x + 1$$

Recall from MAT 1375

$$f(x) = \frac{1}{x-a}, \; a \in \mathbb{R}$$



Left side

$$\lim_{x \to a^-} \frac{1}{x-a} = -\infty$$

Right side

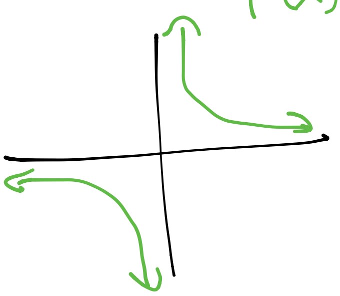
$$\lim_{x \to a^+} \frac{1}{x+a} = \infty$$

horizontal

$\frac{1}{x-a}$ is translation of $\frac{1}{x}$

$f(x-a)$ is $f(x)$ moved a spaces to the right

$f(x) = \frac{1}{x}$



$f(x-a) = \frac{1}{x-a}$