- There are normally three main steps in solving an applied optimization problem:
  (1) **Choose the variables.**
  Determine which quantities are relevant, perhaps by drawing a diagram, and assign to each an appropriate variable name.
  (2) **Find the function and the interval.**
  Restate the question as an optimization problem for a function f over some interval I. If the function depends on more than one variable, use a constraint equation to write f as a function of just one variable.
  (3) **Optimize the function.**
- If the interval I is open, f does not necessarily take on a minimum or maximum value on I. If it does, though, the values must occur at critical points within the interval. To determine whether a minimum or maximum exists, analyze the behavior of F as x approaches the endpoints of the interval.

1. A 100-inch piece of wire is divided into two pieces and each piece is bent into a square. How should this be done in order to minimize the sum of the areas of the two squares?

(a) Express the sum of the areas of the squares in terms of the lengths x and y of the two pieces.

(b) What is the constraint equation relating x and y?

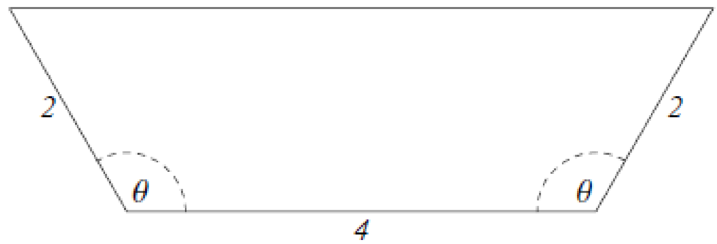
(c) Does this problem require optimization over an open interval or a closed interval?

(d) Solve the optimization problem.

2. The legs of a right triangle have lengths a and b satisfying $a + b = 10$. Which values of a and b maximize the area of the triangle?

3. A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$1 per square foot, and the metal for the sides costs \$2 per square foot. Find the dimensions that minimize cost if the box has to have a volume of 20 ft^3.

4. Find the angle θ that maximizes the area of the trapezoid with base of length 4 and sides of length 2.



5. Find the point P on the parabola y=x^2 closest to the point (3,0). (Figure 12 on p.263 of your textbook.)

6. Find the area of the largest isosceles triangle that can be inscribed in a circle of radius r.

Solution:

1. A 100-inch piece of wire is divided into two pieces and each piece is bent into a square. How should this be done in order to minimize the sum of the areas of the two squares?

Let x and y be the lengths of the pieces.

(a) Express the sum of the areas of the squares in terms of the lengths x and y of the two pieces.

The perimeter of the first square is x, which implies the length of each side is $\frac{x}{4}$ and the area is $\left(\frac{x}{4}\right)^2$. Similarly, the area of the second square is $\left(\frac{y}{4}\right)^2$. Then the sum of the areas is given by $A = \left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2$.

(b) What is the constraint equation relating x and y?

$x + y = 100$, whence $y = 100 - x$. Then

$$A(x) = \left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = \left(\frac{x}{4}\right)^2 + \left(\frac{100-x}{4}\right)^2 = \frac{1}{8}x^2 - \frac{25}{2}x + 625$$

(c) Does this problem require optimization over an open interval or a closed interval?

Since it is possible for the minimum total area to be realized by not cutting the wire at all, optimization over the closed interval $[0, 100]$ suffices.

(d) Solve the optimization problem.

Solve $A'(x) = \frac{1}{4}x - \frac{25}{2} = 0$ to obtain $x = 50$. Now $A(0) = A(100) = 625$, whereas $A(50) = 312.5$. Accordingly, the sum of the areas of the squares is minimized if the wire is cut in half.