Practice Exam 1

Calculus I (Halleck)

- 1. Find the domain and the range of the function $y = \frac{1}{1 + \sqrt{1 x^2}}$.
- 2. Given $f(x) = \sqrt{\frac{x}{1-x}}$ and $g(x) = x^2$. Calculate the composite functions $f \circ g$ and $g \circ f$ and determine their domains.
- 3. Find $\cos \theta$, $\sec \theta$ and $\tan \theta$ if $\sin \theta = \frac{1}{3}$ and θ is obtuse (> $\pi/2$).
- 4. Determine $\csc\left(\tan^{-1}\sqrt{1+x^2}\right)$.
- 5. The volume of a sphere of radius R is $V = \frac{4\pi R^3}{3}$. What is the average rate of change of the volume when the radius increases from R = 1 to R = 3?
- 6. Determine the one-sided limits at C = 0, 5, 6, 7 of the function f(x) shown in the figure and state whether the limit exists at these points (if a limit is infinite, write ∞ or $-\infty$).



7. Determine the points where the function is not continuous and state the type of discontinuity: removable, jump, infinite, or none of these.

A)
$$\frac{x^2 + 2x - 8}{|x - 2|}$$

B)
$$\frac{x}{[x]}, x \ge 1$$

C)
$$\frac{(x^3 - 3x + 2)\sin 2x}{x^3 - 3x + 2}$$

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8. Evaluate the limit or state that it does not exist (you must have evidence and show work to get any credit).

A)
$$\lim_{x \to 2} \frac{x^4 - x^2 - 12}{x - 2}$$

B)
$$\lim_{x \to 1} \frac{\sqrt{x^2 + 1} - \sqrt{x + 1}}{x^2 + x - 2}$$

C)
$$\lim_{x \to 2} \frac{\sqrt{x^2 - 4x + 4}}{x - 2}$$

- 9. Given a function f, what does the quantity $\frac{f(3)-f(1)}{2}$ represent?
- 10. If $\lim_{h \to 0} \frac{f(3+h) f(3)}{h} = \lim_{h \to 0} \frac{f(4+h) f(4)}{h} = 3.5$ then what do you know regarding the derivative of f?
- 11. Let f be a function satisfying $f'(x) = 4x^4 + f^2(x) + 2^x$. Which of the following statements must be true?
 - A) f'(0) = 1
 - B) f(x) > 0 for all x
 - C) The tangent line to the graph of f at every point has a positive slope.
 - D) f is not differentiable at x = 0
- 12. Find coordinates of all points on the curve $y = \sqrt{x} .25x^2 + 4$ at which the slope of the tangent line is horizontal. What is the equation of the tangent line(s)?
- 13. Calculate the derivatives of the following functions:

$$A) \quad g(x) = \frac{\sqrt{x} + e^x}{4x^2 - 3x}$$

B) $h(x) = \left(e^x + 5\right) \left(\frac{x^2 + 4}{x - 1}\right)$

Bonus: Use the limit definition of derivative together with properties of limits to prove the quotient rule.